Study guide for STAT3004/STAT7304, Semester 1, 2020

Last updated May 15, 2020

Here is a list of activities that need to be carried out for self study.

Activity 1

READ All of [SP-1-Section 1.1] Definitions and Examples.

Focus on understanding examples:

- 1.2 - Ehrenfest chain
- 1.6 - Inventory chain
- 1.8 - Branching processes
- 1.10 - Two stage Markov chains

Try to run a Monte Carlo simulation (for some parameters) for each of these.

READ: Section 1.2 Multistep Transition Probabilities.

Try to reproduce the numerical examples in example 1.11 (Gambler's ruin).

READ [MC-1-Example 1.1.5] Virus mutation.

Recall the computation carried out in class (Week 3) for the two state Markov chain to obtain the equation at the bottom of the example for the probability that the strain in the nth generation is the same as at the start.

As described in the example, this result can also be obtained by considering the NxN transition matrix and computing $p_{11}^{(n)}$. Obtain such values numerically (for some alpha) and see the results agree.

EXTRA: Reproduce Example 1.1.6 from [MC-1]

Activity 2

READ [MC-1] Section 1.2 Class structure.

Carry out Exercise 1.2.1 (Which classes are closed?)

READ [SP-1] Section 1.3 Classification of states. Do it twice.

- On the first read skip the theorem proofs.

- On the second read, rewrite (to understand) the theorem proofs (where available).

READ [MC-1] Section 1.3 Hitting times and absorption probabilities

- Follow the proof of Theorem 1.3.2
- Carry out exercise 1.3.4

Activity 3

READ [SP-1] Section 1.4 Stationary Distributions

- Verify some of the numerical examples in the Section via Monte Carlo simulation.

Skim Section 1.4.1 dealing with Doubly Stochastic Chains

Skim Section 1.5 dealing the detailed balance condition

Skim Section 1.5.1 dealing with reversibility

Skip (unless you wish to read) Section 1.5.2 dealing with the Metropolis-Hastings Algorithm

Skip (unless you wish to read) Section 1.5.3 dealing with Kolmogorov Cycle Condition

READ [MC-1] Section 1.4, Strong Markov property

READ [MC-1] Section 1.5, Recurrence and transience

Skim [MC-1] Section 1.6, Recurrence and transience of random walks

READ [SWJ-10], Section 10.2, Markov Chains up to (and not including) Continuous Time Markov Chains

Try to replicate the examples from [SWJ-10]

Activity 4

READ [SP-1] Section 1.6, Limit Behavior

READ [SP-1] Section 1.7, Returns to a Fixed State

Skim [SP-1] Section 1.8, Proof of the Convergence Theorem

Skim [MC-1] Section 1.7, Invariant distributions

Skim [MC-1] Section 1.8, Convegence to equilbrium

Skip (unless you wish to read) [MC-1] Section 1.9, Time reversal

Skim [MC-1] Section 1.10 Ergodic theorem

Skim [SP-1] Section 1.11 Infinite State Spaces

Activity 5

READ [EM-4], Stochastic Models in Discrete Time

Carry out Project 1 focusing on the models from [EM-4]

Activity 6

READ [SP-2] Section 2.1, Exponential Distribution

READ [SP-2] Section 2.2, Defining the Poisson Process

READ [SP-2] Section 2.2.1, Constructing the Poisson Process

Skim [SP-2] Section 2.2.2, More Realistic Models

Skip (unless you wish to read) [SP-2], Section 2.3, Compound Poisson Process

Skim [SP-2] Section 2.4, Transformations

Activity 7 - basics of CTMC

READ [SP-4] Section 4.1, Definitions and Examples

READ [SP-4] Section 4.2, Computing the Transition Probability

READ [SWJ-10], Continuous Time Markov Chains in Section 10.2

- Follow code example 10.6
- Follow code example 10.7

Skim [SWJ-10], Section 10.3, Discrete Event Simulation

READ [MC-2], Section 2.1, Q-matrices and their exponentials

READ [MC-2], Section 2.2, Continuous-time random processes

Skim [SP-4] Section 4.2.1, Branching Processes

READ [SP-4] Section 4.3, Limiting Behavior

Skim [SP-4] Section 4.3.1, Detailed Balance Condition

Skim [SP-4] Section 4.4, Exit Distributions and Exit Times

READ/Skim [SP-4] Section 4.5, Markovian Queues

Skip [SP-4] Section 4.6, Queueing Networks

Activity 8 - Epidemic models in continuous time [EM-3] and is the focus of Project 2

READ [EM-3], Section 3.1, The Simple Stochastic Epidemic in Continuous Time

- This is the stochastic SI model.
- See how it is a pure-death CTMC.

Skim [EM-3], Section 3.1.3, Distribution of the duration time

Skim [EM-3], Section 3.1.2, p.g.f. methods for Markov Chains

READ [EM-3], Section 3.3, The General Stochastic Epidemic

- This is the stochastic SIR model.
- Watch first 20 minutes of this video describing determinstic SIR (https://www.youtube.com/watch?v=uG3ipWMzn5Q (https://www.youtube.com/watch?v=uG3ipWMzn5Q))

Skim [EM-3], Section 3.3.1, Solution by the p.g.f. method

Skim [EM-3], Rest of subsections of Section 3.3

Carry out Project 2.

Activity 9 - Further mathematical details of CTMC

Note: much of the material covered in reading here has already been read in activity 8. Now in [MC] it is more rigorous than in [SP].

Note: In general - there is much reading here - focus should be on the results and intution and some of the proofs may be skipped.

READ [MC-2], Section 2.5, Birth Processes

READ [MC-2], Section 2.6, Jump Chain and holding Times

- This is the "Embedded Markov Chain"

Skim [MC-2], Section 2.7, Explosion

READ [MC-2], Section 2.8, Forward and backard equations

Skip [MC-2], Section 2.9, Non-minimal chains

READ [MC-3], Section 3.1, Basic properties

- This will be a review

Skim [MC-3], Section 3.2, Class structure

- Not much different than discrete time Markov chains.

Skim [MC-3], Section 3.3, Hitting times and absorbtion probabilities

READ [MC-3], Section 3.4, Recurrence and transcience

- Only read theorem statements.

READ [MC-3], Section 3.5, Invariant distributions

- Only read theorem statements.

Skim [MC-3], Section 3.6, Convergence to equilibrium

- Only read theorem statements.

Skip [MC-3], Section 3.7, Time Reversal ¶

Skim [MC-3], Section 3.8, Ergodic theorem

- Only read theorem statements.

Activity 10 - Probability and Measure (Optional)

Skim [MC-6], Sections 6.1 - 6.4 covering basic measure theoretic probability.

Skim [MC-6], Section 6.5, applying to stopping times.