SCIE1000, Computing Tutorial Week 8, Semester 1, 2011.

Aims

The aims of this computing tutorial are to:

- To reinforce many of the Python programming principles that you have met so far in SCIE1000.
- To gain experience at using a numerical algorithm (Newton's method) to solve equations.

Note that this program does not involve using any new Python programming constructs, but instead builds completely on concepts you have covered in recent weeks. Looking at the programs you have already written in computing tutorials will certainly help with developing this program.

1. In a recent lecture and the tutorial this week we used Newton's method as an algorithm for finding an approximate numerical solution to an equation.

   (a) Write a Python program to implement Newton's method to estimate $\sqrt{12}$, as was done on Page 232 of the lecture notes. Start by studying the formal description of the algorithm in the lecture notes: you basically need to "translate" this (English) description of the algorithm into Python. Think about the programming techniques that you have used in previous weeks. Do you need a loop? What variables do you need? Make sure you write your program so that it can be easily modified to solve different functions.

   (b) Run your program. Your result should be identical to the table in the notes (for two iterations). Run your program for a larger number of iterations and observe how the solution value changes.

2. On Pages 244 – 245 of the notes, we used Newton's method to find the time at which the blood concentration of an injected hormonal contraceptive fell below the level required for effective contraceptive function (0.3 ng/mL). Specifically, on Page 245 of the notes we say:

   [The blood concentration at any time t in days is] "$C(t) = 1.4t^{0.15}e^{-0.02t}$. The equation that requires solving is $C(t) = 0.3$. Hence if we let $f(t) = C(t) - 0.3$ then we can solve $f(t) = 0$ by using Newton's method, as follows:

   $$f(t) = 1.4t^{0.15}e^{-0.02t} - 0.3, \quad \text{so}$$

   $$f'(t) = 1.4e^{-0.02t}\left(0.15t^{-0.85} - 0.02t^{0.15}\right)$$

   Finally, we use $t_0 = 50$ as the initial estimate for the solution."

   (a) Modify your program from Question 1 to find an approximate solution to this problem. (You should find that after 4 or 5 steps, the solution is $t \approx 112.44$ days.)

   (b) Now, find the time at which the concentration is 1.5 ng/mL, using an initial estimate of 10 days.

   (c) Repeat Part (b), but use an initial estimate of 1 day.

   (d) Explain your answers to Parts (b) and (c). (Hint: it may help to look at the graph on Page 244 of the lecture notes.)

There are no classes next week. The SCIE1000 midsemester exam will be held on the first week back. Now is a good time to look back at what we have covered so far, and ask your tutor any questions that you might have.

The end