

**SCIE1000 — Theory and Practice in Science**  
**Midsemester Examination, Semester One 2011**

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1. Ecologists are considering two models for the biomass (in tonnes) of fish in a pond:

$$M_1 = 20(1 - e^{-0.2t}) \quad \text{and} \quad M_2 = 5\sqrt{t}$$

where  $t$  is the number of months since the most recent harvest.

- (a) One of the above models includes a maximum “carrying capacity” for the fish (that is, a maximum limit to the population size), and the other model increases indefinitely. Identify which of  $M_1$  and  $M_2$  has the carrying capacity, and explain your answer briefly.

(2 marks)

- (b) Find the time at which  $M_1 = 10$ .

(3 marks)

- (c) Find the time at which  $M_2 = 10$ .

(2 marks)

2. (a) Assume that the biomass of all of the people living on Earth were ‘compact’ into a cube of side  $s$ . Estimate the value of  $s$  **in km**. Show all working, list any values that you have assumed, and include units in your answer. (Hints: assume that there are no ‘air gaps’ or ‘spaces’ within the cube. The average density of a human is about 1000 kg per  $\text{m}^3$ . The volume of cube of side  $s$  is  $s^3$ . There are 1000 L per  $\text{m}^3$ .)

(6 marks)

- (b) Use your approach from Part (a) to write a mathematical expression for the length  $s$  of the side of the cube into which you could pack the biomass of  $N$  people.

(3 marks)

- (c) Sketch a rough graph of  $s$ , showing the shape of the graph as  $N$  increases.

(1 mark)

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3. Three measures of the effectiveness of a test for a given condition are the *sensitivity*, *specificity* and *accuracy* of the test. These are defined as follows, where  $A, B, C$  and  $D$  are the values in the table, and  $N$  is the total population.

• Sensitivity =  $\frac{A}{A + C}$       • Specificity =  $\frac{D}{B + D}$       • Accuracy =  $\frac{A + D}{N}$

		Condition: Is the condition present?	
		Yes	No
Test	Positive	$A$	$B$
	Negative	$C$	$D$

Urine pregnancy tests may be purchased at pharmacies and conducted at home. As a young scientist, you wonder how the urine pregnancy test compares with the “gold standard” serum beta hCG (pregnancy) test. The following study compares the tests.

200 women who thought they may be pregnant underwent both urine and serum hCG pregnancy tests. The results of the blood test indicated that 155 women were pregnant. Of these, 139 had tested positive on the urine test. There was a total of 57 negative urine tests.

(a) Calculate the sensitivity, specificity and accuracy of the urine test. (4 marks)

(b) After undergoing fertility treatment, there is a 20% chance that a woman is pregnant. Find the probability she is pregnant if she returns a positive urine test result?

(4 marks)

4. In lectures, we used the following function  $d(t)$  to model the probability that a woman will die of breast cancer **prior to reaching age  $t$  years**, where  $t$  is between 30 and 85. Note that  $d(t)$  simplifies to:  $d(t) \approx 7.7 \times 10^{-6} \times (t - 30)^2$ .

$$d(t) = \frac{1}{43} \times \frac{1}{55^2} \times (t - 30)^2.$$

- (a) In the box, write all of the output from the following partial program. (6 marks)

```
a=zeros(4)
d=zeros(4)
i=0
while i<4:
    a[i] = (i+4) * 10
    d[i] = 0.77 * (a[i]-30)**2
    print a[i],d[i]
    i = i+1
```

The output is:

- (b) The program in Part (a) is intended to evaluate  $d(t)$  per million women (for various ages  $t$ ). However, it does not produce correct output. Briefly explain the error(s) in the program. (1 mark)

*(continued over)*

(c) Recall that

$$d(t) = \frac{1}{43} \times \frac{1}{55^2} \times (t - 30)^2.$$

Consider a different population of women in which the probability of dying of breast cancer still follows a quadratic function, but deaths from cancer commence at age 25, the life expectancy of women is 90, and the life-long risk of death caused by breast cancer is 4%. Modify  $d(t)$  to represent the new population.

(4 marks)

5. Find an equation that models the following graph:

(4 marks)

