SCIE1000, Tutorial Week 11

- This week you will mostly work through questions from previous exam papers.
- As usual, you should recognise that the broad concepts and techniques we cover are more important than the specific examples. Do not try to commit lots of facts to memory; instead, know **how** to do things, and **when** certain models and approaches are appropriate.
- You should have started preparing for the final exam. Remember that it is open-book. What materials will you take in? Do you need to re-write any key points in short, easily accessible form? There are no memory questions on the exam, so don't try to comit things to memory.
- Have a look at previous papers, particularly from Semester 1, 2010. Could you answer those questions, in 2 hours, if you didn't know what any of them were going to be? If not, then practise!

1 Questions

1. (Final exam, 2010.) The *Glycaemic Index* (GI) of a food is defined by

$$GI = 100 \times \frac{\int_0^2 f(t)\, dt}{\int_0^2 g(t)\, dt}$$

where t is measured in hours and

- $f(t)$ is the **increase** in blood glucose concentration (compared with the fasting level) after consuming a controlled dose of that food
- $g(t)$ is the **increase** in blood glucose concentration (compared with the fasting level) after consuming a controlled dose of glucose.

(a) (Worth 4 marks so about 4 minutes to work.) A person's fasting glucose level is 4 mmol L^{-1}. Their **total** blood glucose level for a period of 2 hours after consuming glucose at time $t = 0$ is $-4t^2+8t+4$ mmol L^{-1}. Find $g(t)$ and hence find $\int_0^2 g(t)\, dt$.

(Hint: Note that $g(t)$ is the **increase** in blood concentration over the fasting glucose level. Also, $\int at^2 + bt + c\, dt = \frac{at^3}{3} + \frac{bt^2}{2} + ct + d$, where a, b, c, d are constants.)

(b) (Worth 6 marks so about 6 minutes to work.) The person in Part (a) consumes a controlled dose of food, giving the following **increase in blood glucose over the fasting level**.

Time (hours)	0	0.3	0.6	1.0	1.5	2.0
Increased level (mmol L^{-1})	0	2	3	3	2	1

Recall that $f(t)$ is the **increase** in blood glucose concentration. Use areas of rectangles to estimate $\int_0^2 f(t)\, dt$.

(c) (Worth 1 mark so about 1 minute to work.) What are the units of the value you calculated in Part (b)?

(d) (Worth 1 mark so about 1 minute to work.) Recall that $GI = 100 \times \frac{\int_0^2 f(t)\,dt}{\int_0^2 g(t)\,dt}$. Use your answers to Parts (a) and (b) to estimate the GI of the food.

2. Bob the biologist is modelling the growth of a certain species of algae over a given time period. Let $P(t)$ be the population of algae at any time t in hours, in individuals per mL of water.

   (a) Bob believes that the population satisfies the differential equation $P' = kP$, where k is a constant. Explain briefly, in words, what this equation means. What is the physical meaning of the constant k?

   (b) Show that $P(t) = Ae^{kt}$ is a solution to the equation in Part (a), where A is a constant. What is the physical meaning of the constant A?

   (c) Recall that $P(t) = Ae^{kt}$. Bob's experiments show that at time $t = 2$ hours, $P(2) = 200$ individuals per mL of water, and at time $t = 6$ hours, $P(6) = 400$ individuals per mL. Find an equation for the population $P(t)$. (Round the value of the constant A to zero decimal places, and the value of k to three decimal places.)

   (d) Bob asks you whether his model for $P(t)$ in Part (c) is likely to be realistic over an extended time period. Respond to Bob's question, with reasons justifying your answer. (You should include a rough sketch of the algae population over time as predicted by Bob's model. If you believe his model is inaccurate, include a rough sketch of what you believe is a more accurate prediction of the population over time.)

3. (Final exam, 2010. Worth 15 marks, so about 15 minutes to work.) The *von Bertalanffy growth model* states that the rate of increase of the length $L(t)$ of a shark of age t in years is proportional to an intrinsic positive growth rate r and the difference between a fixed maximum length M and its current length $L(t)$.

   (a) Write a differential equation (DE) for the length of the shark at any time.
   (Hint: your answer should be of the form $L'(t) = \ldots$.)

   (b) Show that $L(t) = M - (M - L_0)e^{-rt}$ is a solution to the DE in Part (a), where L_0 is the length of the shark at time $t = 0$ when it is born.
   (Hint: if $y(t) = e^{-rt}$ then $y'(t) = -re^{-rt}$.)

   (c) Recall that a solution to the DE in Part (a) is $L(t) = M - (M - L_0)e^{-rt}$. For a particular shark, $M = 3$ m, $L_0 = 0.5$ m, t is measured in years and $r = 0.15$ per year.

   (i) Find the time at which the shark reaches 2 m in length.

   (ii) Draw a rough sketch of the length of the shark, for values of t between 0 and 30.

4. (Special exam, 2010. Worth 7 marks so about 7 minutes to work.) Estimate the number of hairs on a "typical" adult human (include the entire person). **Use units in your calculations and clearly state any values you assume.**

The end