# Some factorizations of graphs into $C_{5}$-Factors and 1-Factors 

University of Queensland Research Report<br>Peter Adams and Darryn Bryant<br>Department of Mathematics<br>The University of Queensland<br>Qld 4072, Australia<br>Saad I. El-Zanati<br>Department of Mathematics<br>Illinois State University<br>Normal, Illinois 61790-4520, USA<br>Heather Gavlas*<br>Department of Mathematics \& Statistics<br>University of Vermont<br>16 Colchester Ave.<br>Burlington, Vermont 05401-0156, USA

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In [1], we show that $K_{10 n}$ can be factored into $\alpha C_{5}$-factors and $\beta 1$-factors for all nonnegative integers $\alpha$ and $\beta$ satisfying $2 \alpha+\beta=10 n-1$. The constructions in [1] rely on the existence of factorizations of certain graphs into $C_{5}$-factors and 1-factors. In this research report we present the necessary factorizations, which were found computationally.
[1] Peter Adams, Darryn E. Bryant, Saad I. El-Zanati and Heather Gavlas, Factorizations of the Complete Graph into $C_{5}$-Factors and 1-Factors, (submitted).
[2] D.G. Hoffman and P.J. Schellenberg, The existence of $C_{k}$-factorizations of $K_{2 n}-F$, Discrete Math., 97 (1991) 243-250.
[3] Jiuqiang Liu, A generalization of the Oberwolfach problem and $C_{t}$-factorizations of complete equipartite graphs, J. Combin. Designs, 8 (2000) 42-49.

[^0]In this section, we show that the graphs $K_{10}, K_{20}, K_{3(10)}$, and $K_{4(10)}$ are completely $\left\{C_{5}, K_{2}\right\}$-factorizable, that is, we show that for each $d$-regular graph $G \in\left\{K_{10}, K_{20}, K_{3(10)}\right.$, $\left.K_{4(10)}\right\}, G=\alpha C_{5}+\beta K_{2}$ for all non-negative integers $\alpha, \beta$ satisfying $2 \alpha+\beta=d$. We also give a factorization of $K_{3(4)}$ required in [1].
$K_{10}$ Let the vertex set be $\mathbb{Z}_{10}$. First, $K_{10}=9 K_{2}$ since a 1-factorization of $K_{10}$ exists and $K_{10}=4 C_{5}+K_{2}$ by [2]. For $1 \leq \alpha \leq 3$, the factorizations are:

$$
\begin{array}{lll}
\alpha=1, \beta=7: & ((0,1,2,3,4),(5,6,7,8,9)) & ((0,2),(1,3),(4,5),(6,8),(7,9)) \\
& ((0,3),(1,4),(2,7),(5,8),(6,9)) & ((0,5),(1,6),(2,8),(3,7),(4,9)) \\
& ((0,6),(1,5),(2,9),(3,8),(4,7)) & ((0,7),(1,8),(2,5),(3,9),(4,6)) \\
& ((0,8),(1,9),(2,4),(3,6),(5,7)) & ((0,9),(1,7),(2,6),(3,5),(4,8)) \\
\alpha=2, \beta=5: & ((0,1,3,6,8),(2,9,4,5,7)) & ((1,2,4,7,8),(0,3,9,5,6)) \\
& ((0,4),(1,5),(2,6),(3,7),(8,9)) & ((0,5),(1,7),(2,3),(4,8),(6,9)) \\
& ((0,7),(1,9),(2,5),(3,8),(4,6)) & ((0,2),(1,6),(3,4),(5,8),(7,9)) \\
& ((0,9),(1,4),(2,8),(3,5),(6,7)) & \\
\alpha=3, \beta=3: & ((8,0,1,3,6),(9,4,5,7,2)) & ((8,1,2,4,7),(9,5,6,0,3)) \\
& ((9,0,5,3,7),(8,4,1,6,2)) & ((0,2),(1,9),(3,4),(5,8),(6,7)) \\
& ((0,7),(1,5),(2,3),(4,6),(8,9)) & ((0,4),(1,7),(2,5),(3,8),(6,9))
\end{array}
$$

$K_{20}$ Let the vertex set of $K_{20}$ be $\mathbb{Z}_{20}$. Since $K_{20}=K_{10}+K_{10,10}=\alpha C_{5}+\beta K_{2}+10 K_{2}=\alpha C_{5}+(\beta+10) K_{2}$ for all $\alpha, \beta$ with $2 \alpha+\beta=9$, it follows that $K_{20}=\alpha C_{5}+\beta K_{2}$ for $(\alpha, \beta) \in\{(0,19),(1,17),(2,15),(3,13)$, $(4,14)\}$. Also $K_{20}=9 C_{5}+K_{2}$ by [2]. For $5 \leq \alpha \leq 9$, the factorizations are:

$$
\begin{aligned}
& \alpha=5, \beta=9: \\
& ((0,10,1,11,12),(2,8,16,3,14),(4,15,6,9,18),(5,17,7,13,19)) \\
& ((0,11,4,8,17),(1,13,6,12,19),(2,16,9,3,18),(5,10,14,7,15)) \\
& ((0,13,11,6,16),(1,14,5,8,18),(2,17,9,4,19),(3,12,7,10,15)) \\
& ((0,14,6,8,19),(1,15,11,7,16),(2,9,5,18,12),(3,13,10,4,17)) \\
& ((0,15,2,13,18),(1,12,10,6,17),(3,8,7,9,19),(4,14,11,5,16)) \\
& ((0,1),(2,3),(4,5),(6,7),(8,9),(10,11),(12,13),(14,15),(16,17),(18,19)) \\
& ((0,2),(1,3),(4,6),(5,7),(8,10),(9,11),(12,14),(13,15),(16,18),(17,19)) \\
& ((0,3),(1,2),(4,7),(5,6),(8,11),(9,10),(12,15),(13,14),(16,19),(17,18)) \\
& ((0,4),(1,5),(2,6),(3,7),(8,12),(9,13),(10,16),(11,17),(14,18),(15,19)) \\
& ((0,5),(1,4),(2,7),(3,6),(8,13),(9,12),(10,17),(11,16),(14,19),(15,18)) \\
& ((0,6),(1,7),(2,4),(3,5),(8,14),(9,15),(10,18),(11,19),(12,16),(13,17)) \\
& ((0,7),(1,6),(2,5),(3,4),(8,15),(9,14),(10,19),(11,18),(12,17),(13,16)) \\
& ((0,8),(1,9),(2,10),(3,11),(4,12),(5,13),(6,18),(7,19),(14,16),(15,17)) \\
& ((0,9),(1,8),(2,11),(3,10),(4,13),(5,12),(6,19),(7,18),(14,17),(15,16)) \\
& \\
& \alpha=6, \beta=7: \\
& ((0,8,1,9,16),(2,10,3,11,12),(4,13,18,5,14),(6,15,17,7,19)) \\
& ((0,9,2,8,17),(1,10,4,11,13),(3,12,19,5,15),(6,14,16,7,18)) \\
& ((0,10,12,1), 14),(2,11,5,13,19),(3,8,7,15,16),(4,17,6,9,18)) \\
& ((0,11,15,1,18),(2,13,3,9,17),(4,16,6,8,19),(5,10,14,7,12)) \\
& ((0,12,18,2,15),(1,17,14,3,19),(4,8,16,5,9),(6,11,7,10,13)) \\
& ((0,13,7,9,19),(1,11,14,2,16),(3,17,5,8,18),(4,12,6,10,15)) \\
& ((0,1),(2,3),(4,5),(6,7),(8,9),(10,11),(12,13),(14,15),(16,17),(18,19)) \\
& ((0,2),(1,3),(4,6),(5,7),(8,10),(9,11),(12,14),(13,15),(16,18),(17,19)) \\
& ((0,3),(1,2),(4,7),(5,6),(8,11),(9,10),(12,15),(13,14),(16,19),(17,18)) \\
& ((0,4),(1,5),(2,6),(3,7),(8,12),(9,13),(10,16),(11,17),(14,18),(15,19))
\end{aligned}
$$

$((0,5),(1,4),(2,7),(3,6),(8,13),(9,12),(10,17),(11,16),(14,19),(15,18))$ $((0,6),(1,7),(2,4),(3,5),(8,14),(9,15),(10,18),(11,19),(12,16),(13,17))$ $((0,7),(1,6),(2,5),(3,4),(8,15),(9,14),(10,19),(11,18),(12,17),(13,16))$
$\alpha=7, \beta=5$ :
$((0,2,1,3,4),(5,6,8,7,10),(9,11,12,14,17),(13,15,16,18,19))$
$((0,5,1,4,6),(2,7,3,9,10),(8,13,14,11,15),(12,18,17,16,19))$
$((0,7,11,1,8),(2,4,9,5,15),(3,13,18,14,19),(6,12,17,10,16))$
$((0,9,1,10,14),(2,5,16,7,13),(3,12,15,4,17),(6,11,19,8,18))$
$((0,10,12,7,19),(1,15,6,2,17),(3,11,13,9,16),(4,8,14,5,18))$
$((0,13,5,11,17),(1,16,12,4,19),(2,8,10,3,18),(6,9,15,7,14))$
$((0,15,10,4,16),(1,13,6,3,14),(2,9,19,5,12),(7,17,8,11,18))$
$((19,10),(7,9),(6,17),(16,13),(12,1),(5,8),(4,14),(0,18),(11,2),(15,3))$
$((18,9),(16,11),(7,4),(3,8),(6,1),(12,0),(17,5),(19,2),(10,13),(15,14))$
$((12,8),(7,1),(19,6),(17,15),(13,4),(0,11),(9,14),(5,3),(2,16),(10,18))$
$((8,16),(13,17),(3,0),(6,10),(19,15),(9,12),(11,4),(7,5),(2,14),(1,18))$
$((0,1),(2,3),(4,5),(6,7),(8,9),(10,11),(12,13),(14,16),(15,18),(17,19))$
$\alpha=8, \beta=3$ :
$((0,2,1,4,3),(5,6,9,8,7),(10,12,14,11,16),(13,17,18,15,19))$
$((0,4,2,5,9),(1,6,3,7,11),(8,10,13,14,18),(12,17,15,16,19))$
$((0,5,1,7,12),(2,6,10,3,16),(4,8,17,14,19),(9,11,15,13,18))$
$((0,6,11,2,7),(1,8,13,3,15),(4,17,9,12,18),(5,16,14,10,19))$
$((0,10,1,9,13),(2,12,15,4,14),(3,8,16,6,19),(5,11,18,7,17))$
$((0,11,8,12,16),(1,14,5,3,17),(2,9,15,7,19),(4,10,18,6,13))$
$((0,15,5,10,17),(1,18,2,8,19),(3,9,14,6,12),(4,7,16,13,11))$
$((0,18,3,11,19),(1,12,4,9,16),(2,10,15,6,17),(5,8,14,7,13))$
$((19,9),(8,0),(1,13),(10,7),(12,11),(15,2),(16,17),(14,3),(4,6),(5,18))$
$((8,15),(14,0),(13,2),(4,16),(1,3),(12,5),(7,6),(17,11),(19,18),(10,9))$
$((0,1),(2,3),(4,5),(6,8),(7,9),(10,11),(12,13),(14,15),(16,18),(17,19))$
$K_{3(10)}$ Let the vertex set of $K_{3(10)}$ be $\mathbb{Z}_{30}$ with the vertex partition $\{0,1, \ldots, 9\},\{10,11, \ldots, 19\}$, and $\{20,21, \ldots, 29\}$. For all possible values of $\alpha$, the factorizations are:

$$
\begin{aligned}
& \alpha=0, \beta=20 \text { : } \\
& ((28,7),(25,12),(19,3),(8,21),(16,22),(2,14),(13,5),(29,4),(24,1),(9,18) \text {, } \\
& (20,11),(15,6),(10,23),(0,27),(17,26)) \\
& ((22,0),(11,24),(15,9),(25,10),(16,7),(4,21),(23,8),(27,13),(5,17),(14,29) \text {, } \\
& (1,28),(6,19),(20,12),(18,3),(26,2)) \\
& ((12,9),(7,25),(17,4),(16,29),(28,11),(20,15),(27,14),(22,10),(6,26),(23,3) \text {, } \\
& (13,8),(1,21),(18,2),(19,5),(0,24)) \\
& ((11,1),(29,15),(18,4),(6,23),(0,28),(7,12),(19,20),(13,2),(10,5),(21,16) \text {, } \\
& (26,3),(27,9),(25,8),(24,14),(22,17)) \\
& ((25,11),(5,14),(19,1),(20,18),(21,9),(4,15),(29,17),(10,2),(16,3),(28,12) \text {, } \\
& (26,0),(7,22),(6,27),(8,24),(23,13)) \\
& ((13,29),(20,2),(15,3),(8,26),(6,28),(21,18),(12,23),(11,4),(16,9),(27,17) \text {, } \\
& (1,25),(0,14),(5,24),(22,19),(7,10)) \\
& ((12,2),(22,9),(15,27),(28,18),(10,0),(29,11),(24,17),(3,14),(7,26),(13,4) \text {, } \\
& (6,25),(20,8),(21,19),(23,1),(16,5)) \\
& ((2,16),(21,6),(25,0),(3,12),(5,26),(14,28),(11,23),(8,22),(19,4),(9,29) \text {, } \\
& (15,7),(17,20),(13,1),(10,24),(18,27)) \\
& ((24,3),(0,18),(1,16),(27,4),(21,13),(7,19),(20,10),(11,22),(23,9),(8,15),
\end{aligned}
$$

$(17,25),(14,26),(2,29),(12,6),(5,28))$
$((16,23),(3,13),(8,11),(15,2),(28,10),(1,27),(6,22),(24,12),(25,19),(7,14)$,
$(5,18),(20,4),(9,26),(29,0),(17,21))$
$((17,0),(4,24),(21,10),(23,18),(20,5),(15,26),(2,22),(9,11),(12,1),(6,14)$,
$(27,7),(25,13),(28,3),(8,16),(19,29))$
$((15,5),(0,20),(9,17),(10,1),(8,19),(24,6),(11,21),(18,22),(16,4),(26,12)$,
$(29,7),(27,2),(13,28),(3,25),(14,23))$
$((29,3),(0,13),(27,11),(6,20),(23,5),(2,17),(15,24),(25,4),(28,16),(22,14)$,
$(7,21),(10,9),(8,12),(18,1),(26,19))$
$((22,5),(24,7),(28,15),(6,10),(1,14),(27,19),(23,2),(29,18),(21,3),(17,8)$,
$(12,4),(25,9),(13,20),(16,0),(26,11))$
$((26,16),(23,4),(15,21),(10,27),(9,24),(20,7),(19,28),(3,17),(18,8),(5,29)$,
$(22,1),(25,14),(2,11),(13,6),(0,12))$
$((0,11),(26,10),(24,19),(7,13),(3,27),(17,23),(16,25),(8,29),(14,20),(15,1)$,
$(6,18),(28,9),(22,4),(5,12),(21,2))$
$((24,13),(4,14),(25,2),(5,11),(8,27),(12,21),(26,18),(15,0),(22,3),(23,7)$,
$(16,6),(1,20),(29,10),(17,28),(9,19))$
$((2,19),(4,28),(26,1),(24,18),(20,16),(25,15),(23,0),(21,14),(11,3),(17,7)$,
$(22,12),(8,10),(29,6),(5,27),(13,9))$
$((15,23),(17,6),(24,16),(20,9),(11,7),(22,13),(8,14),(3,10),(12,27),(0,19)$,
$(25,18),(2,28),(5,21),(4,26),(1,29))$
$((21,0),(20,3),(14,9),(26,13),(25,5),(6,11),(29,12),(8,28),(1,17),(10,4)$,
$(22,15),(7,18),(27,16),(2,24),(19,23))$
$\alpha=1, \beta=18:$
$((27,2,11,9,16),(13,25,4,10,28),(17,6,19,23,8),(0,18,21,3,22),(1,26,7,12,24),(29,14,5,20,15))$ $((16,25),(0,14),(19,22),(23,4),(10,3),(21,13),(7,20),(18,27),(2,28),(17,9)$,
$(8,29),(12,6),(15,1),(24,11),(26,5))$
$((6,23),(14,26),(24,5),(7,21),(22,12),(19,25),(15,3),(1,18),(8,28),(2,17)$,
$(13,27),(11,20),(9,10),(29,4),(0,16))$
$((12,3),(2,21),(22,16),(0,25),(29,11),(19,27),(14,28),(26,13),(20,1),(10,7)$,
$(5,18),(9,23),(24,6),(17,4),(15,8))$
$((4,24),(7,23),(22,5),(28,0),(6,26),(27,11),(16,20),(21,14),(8,13),(18,2)$,
$(3,17),(25,15),(12,1),(10,29),(9,19))$
$((1,27),(3,11),(19,8),(12,28),(21,9),(13,29),(7,15),(25,2),(18,20),(10,22)$,
$(6,16),(5,23),(0,17),(4,26),(14,24))$
$((15,22),(13,3),(17,24),(20,10),(26,2),(29,12),(0,19),(9,27),(16,4),(7,11)$,
$(1,23),(25,8),(5,21),(18,28),(14,6))$
$((1,13),(0,10),(6,25),(9,20),(14,2),(19,7),(22,11),(29,16),(21,12),(4,18)$,
$(28,3),(5,27),(26,17),(23,15),(8,24))$
$((26,19),(0,21),(15,2),(9,22),(11,5),(27,3),(18,8),(14,1),(6,13),(7,25)$,
$(24,10),(12,23),(4,20),(29,17),(16,28))$
$((20,12),(19,21),(24,3),(10,8),(2,23),(1,25),(15,4),(9,14),(13,0),(29,6)$,
$(16,7),(22,18),(27,17),(11,26),(28,5))$
$((0,15),(25,9),(1,22),(14,7),(17,5),(4,12),(18,24),(20,13),(10,23),(16,26)$,
$(3,19),(28,6),(27,8),(11,21),(2,29))$
$((9,15),(26,3),(8,11),(25,14),(17,28),(22,13),(19,1),(27,12),(23,18),(20,2)$,
$(7,24),(29,0),(16,5),(21,4),(10,6))$
$((2,10),(29,3),(15,28),(4,22),(23,17),(18,26),(9,12),(21,6),(25,11),(7,27)$, $(13,24),(16,1),(5,19),(8,14),(20,0))$
$((10,26),(19,2),(24,0),(23,11),(9,18),(25,17),(16,3),(21,1),(8,12),(22,6)$,
$(4,28),(15,27),(20,14),(7,13),(5,29))$
$((22,8),(2,24),(7,28),(16,21),(0,12),(10,1),(17,20),(29,9),(23,3),(26,15)$,
$(14,27),(5,13),(19,4),(25,18),(11,6))$
$((24,16),(2,22),(8,20),(4,27),(26,0),(25,3),(18,7),(9,13),(14,23),(12,5)$,
$(11,28),(19,29),(21,10),(15,6),(17,1))$
$((23,16),(15,5),(19,20),(21,17),(27,6),(14,3),(13,4),(1,28),(0,11),(9,24)$,
$(8,26),(18,29),(2,12),(7,22),(25,10))$
$((2,16),(23,13),(12,25),(11,4),(22,14),(7,17),(26,9),(29,1),(15,24),(3,20)$,
$(19,28),(21,8),(27,0),(6,18),(10,5))$
$((0,23),(1,11),(2,13),(3,18),(4,14),(5,25),(6,20),(7,29),(8,16),(9,28)$,
$(10,27),(12,26),(15,21),(17,22),(19,24))$
$\alpha=2, \beta=16$ :
$((16,3,17,8,27),(22,7,15,20,9),(12,24,14,25,5),(28,6,23,19,4),(2,11,29,1,18),(10,21,13,0,26))$ $((4,13,6,29,14),(9,15,22,11,23),(7,10,20,8,26),(18,0,16,1,25),(24,19,27,5,17),(28,2,21,12,3))$ $((3,19),(26,2),(25,11),(8,22),(27,18),(17,1),(9,12),(13,5),(14,21),(10,4)$,
$(23,7),(28,16),(29,15),(6,24),(0,20))$
$((18,20),(2,19),(12,27),(11,5),(15,3),(16,22),(13,8),(10,29),(6,21),(23,1)$,
$(14,7),(17,28),(26,4),(24,9),(0,25))$
$((18,24),(10,0),(27,4),(26,15),(5,22),(23,14),(1,12),(2,29),(25,7),(20,17)$,
$(13,3),(8,11),(9,21),(19,28),(16,6))$
$((11,21),(22,13),(1,24),(28,10),(25,3),(14,0),(4,23),(15,5),(18,26),(29,17)$,
$(27,7),(8,16),(12,6),(9,19),(2,20))$
$((3,18),(29,9),(27,13),(16,7),(11,0),(15,4),(20,12),(22,14),(6,19),(24,8)$,
$(2,23),(5,28),(26,17),(10,25),(21,1))$
$((1,11),(12,7),(23,13),(14,3),(22,18),(2,27),(29,8),(21,15),(19,26),(25,4)$,
$(10,6),(17,0),(16,24),(5,20),(9,28))$
$((24,7),(6,25),(16,2),(26,11),(17,23),(12,8),(27,10),(29,3),(22,4),(13,28)$,
$(9,18),(19,1),(15,0),(20,14),(5,21))$
$((10,5),(23,0),(1,15),(9,27),(14,28),(11,4),(16,21),(19,29),(26,6),(12,22)$,
$(3,20),(2,17),(24,13),(7,18),(8,25))$
$((25,15),(8,14),(28,0),(17,9),(16,23),(12,4),(24,3),(6,20),(22,10),(7,11)$,
$(18,29),(13,2),(1,27),(26,5),(19,21))$
$((22,17),(8,21),(10,9),(19,20),(0,24),(27,15),(12,2),(14,5),(1,26),(23,3)$,
$(29,4),(18,28),(25,16),(11,6),(13,7))$
$((23,8),(19,25),(11,9),(28,12),(26,14),(21,17),(24,15),(3,22),(2,10),(29,0)$,
$(6,27),(1,13),(4,18),(7,20),(16,5))$
$((16,20),(17,6),(10,3),(7,29),(8,18),(11,24),(25,9),(13,26),(23,12),(2,14)$,
$(28,15),(21,4),(1,22),(27,0),(19,5))$
$((10,24),(3,21),(26,9),(15,23),(11,27),(16,4),(20,13),(18,6),(25,12),(17,7)$,
$(19,0),(2,22),(28,8),(14,1),(29,5))$
$((19,7),(11,20),(10,8),(4,17),(2,15),(14,9),(16,26),(5,24),(0,21),(18,23)$,
$(28,1),(6,22),(13,25),(27,3),(12,29))$
$((0,12),(1,20),(2,25),(3,26),(4,24),(5,18),(6,14),(7,21),(8,15),(9,13)$,
$(10,23),(11,28),(16,29),(17,27),(19,22))$
$((0,22),(1,10),(2,24),(3,11),(4,20),(5,23),(6,15),(7,28),(8,19),(9,16)$,
$(12,26),(13,29),(14,27),(17,25),(18,21))$
$\alpha=3, \beta=14$ :
$((10,29,15,2,21),(5,19,3,14,28),(26,4,27,1,12),(22,11,6,25,8),(24,9,18,23,13),(7,17,0,20,16))$ $((3,22,7,25,12),(21,9,23,11,1),(16,28,19,2,27),(26,15,4,24,10),(14,20,6,13,0),(18,5,17,8,29))$ $((0,16,9,14,27),(29,6,19,20,4),(28,10,3,25,2),(12,23,15,24,5),(22,17,1,26,18),(7,11,21,8,13))$ $((21,3),(0,24),(26,19),(29,14),(17,20),(12,7),(13,1),(10,5),(4,23),(15,22)$,
$(2,11),(16,25),(6,27),(9,28),(8,18))$
$((22,0),(21,5),(17,6),(19,25),(7,15),(12,24),(23,3),(27,13),(9,11),(16,26)$,
$(18,1),(28,4),(20,10),(2,29),(14,8))$
$((6,16),(8,28),(15,9),(1,14),(25,5),(19,22),(4,10),(11,26),(20,7),(12,29)$,
$(18,27),(0,23),(24,3),(2,17),(13,21))$
$((12,20),(22,5),(29,1),(27,15),(19,0),(18,24),(16,4),(17,23),(7,21),(14,26)$,
$(8,10),(6,28),(25,9),(2,13),(3,11))$
$((6,15),(19,23),(13,22),(9,17),(18,20),(16,24),(11,8),(1,25),(29,7),(21,4)$,
$(3,26),(0,10),(2,14),(12,28),(27,5))$
$((5,20),(3,29),(19,4),(18,7),(28,17),(25,13),(9,27),(2,26),(14,22),(16,1)$,
$(12,6),(0,11),(21,15),(10,23),(24,8))$
$((0,21),(20,3),(2,10),(9,19),(6,26),(5,23),(12,4),(28,18),(27,8),(22,16)$,
$(7,14),(13,29),(15,1),(24,17),(25,11))$
$((11,29),(14,23),(22,10),(24,7),(3,17),(9,20),(15,5),(28,1),(12,0),(16,8)$,
$(19,27),(21,6),(18,2),(13,26),(4,25))$
$((13,5),(28,15),(12,2),(23,6),(20,1),(22,4),(29,16),(10,7),(9,26),(3,27)$,
$(11,24),(18,0),(19,8),(21,14),(25,17))$
$((23,7),(22,6),(21,12),(28,3),(25,10),(20,13),(4,18),(29,9),(2,16),(19,1)$,
$(27,17),(24,14),(26,8),(5,11),(0,15))$
$((27,11),(25,14),(19,21),(1,23),(18,3),(9,10),(2,22),(24,6),(15,20),(8,12)$,
$(26,0),(17,29),(16,5),(4,13),(7,28))$
$((13,28),(17,4),(2,24),(5,26),(18,21),(6,14),(16,3),(11,20),(1,10),(23,8)$,
$(29,0),(22,9),(19,7),(27,12),(25,15))$
$((28,11),(19,24),(4,14),(21,17),(10,27),(25,0),(13,3),(7,26),(12,9),(16,23)$,
$(18,6),(20,2),(8,15),(5,29),(22,1))$
$((0,28),(1,24),(2,23),(3,15),(4,11),(5,14),(6,10),(7,27),(8,20),(9,13)$,
$(12,22),(16,21),(17,26),(18,25),(19,29))$
$\alpha=4, \beta=12$ :
$((27,13,21,18,7),(6,26,2,28,14),(12,29,10,24,4),(0,19,8,15,22),(17,5,11,1,25),(20,9,16,23,3))$ $((2,16,29,5,12),(21,8,20,7,11),(10,1,14,25,9),(17,0,28,19,27),(13,26,4,22,6),(23,18,24,3,15))$ $((10,26,16,22,5),(14,2,11,20,0),(17,4,21,19,3),(27,15,9,24,8),(13,23,7,12,25),(18,6,29,1,28))$ $((7,10,8,17,21),(16,20,5,23,0),(1,15,28,6,24),(19,2,29,13,9),(3,25,18,22,12),(26,11,27,4,14))$ $((23,19),(7,14),(1,22),(9,11),(12,27),(25,8),(21,0),(26,5),(20,6),(4,16)$,
$(18,3),(13,28),(10,2),(15,29),(24,17))$
$((18,9),(25,5),(29,11),(4,13),(27,14),(19,20),(16,1),(28,17),(10,22),(8,12)$,
$(2,15),(6,23),(0,24),(3,21),(26,7))$
$((15,5),(8,22),(3,27),(1,23),(13,0),(21,9),(14,20),(4,11),(24,7),(26,12)$,
$(18,2),(16,6),(29,17),(10,28),(25,19))$
$((5,13),(21,15),(29,14),(11,8),(7,22),(27,10),(23,12),(25,0),(18,26),(9,28)$,
$(6,19),(17,1),(16,3),(4,20),(2,24))$
$((0,12),(29,4),(19,24),(8,28),(23,9),(15,26),(16,25),(17,2),(18,5),(7,13)$,
$(21,1),(6,27),(14,3),(10,20),(22,11))$
$((16,8),(3,26),(7,25),(24,15),(4,28),(22,14),(11,23),(12,21),(19,29),(20,2)$,
$(27,5),(18,0),(13,1),(9,17),(10,6))$
$((0,10),(26,8),(2,22),(14,5),(20,1),(23,17),(29,3),(25,6),(15,7),(13,24)$,
$(9,12),(11,28),(4,19),(21,16),(27,18))$
$((18,8),(5,21),(17,22),(16,24),(0,26),(28,3),(9,29),(25,10),(11,6),(4,15)$,
$(20,13),(2,27),(7,19),(23,14),(1,12))$
$((18,29),(10,3),(20,15),(19,1),(21,6),(25,11),(12,28),(17,26),(13,2),(22,9)$,
$(14,8),(4,23),(16,7),(5,24),(27,0))$
$((6,12),(21,10),(16,27),(1,18),(29,7),(19,26),(9,14),(28,5),(8,13),(25,4)$,
$(0,15),(23,2),(22,3),(20,17),(24,11))$
$((13,22),(16,28),(7,17),(8,23),(26,1),(14,21),(25,2),(20,18),(5,19),(6,15)$,
$(9,27),(11,3),(4,10),(24,12),(0,29))$
$((0,11),(1,27),(2,21),(3,13),(4,18),(5,16),(6,17),(7,28),(8,29),(9,26)$,
$(10,23),(12,20),(14,24),(15,25),(19,22))$
$\alpha=5, \beta=10$ :
$((5,25,2,23,12),(6,22,19,21,16),(13,20,4,11,24),(17,1,14,28,7),(26,9,18,0,10),(27,3,15,29,8))$ $((10,25,17,5,22),(2,14,24,12,26),(16,9,13,0,20),(4,18,28,1,15),(6,19,29,7,27),(23,3,11,21,8))$ $((20,3,25,7,10),(27,11,26,4,16),(18,21,5,28,6),(1,23,13,8,12),(15,2,29,17,24),(14,22,9,19,0))$ $((11,23,10,5,29),(2,19,28,16,24),(3,26,0,27,17),(25,12,4,21,9),(22,1,13,6,15),(8,18,20,7,14))$ $((0,25,8,26,15),(16,2,21,14,29),(11,6,17,4,28),(5,24,7,13,27),(12,22,18,23,9),(19,3,10,1,20))$ $((16,8),(22,11),(14,26),(29,4),(27,12),(17,9),(20,5),(19,23),(15,7),(3,13)$,
$(21,6),(2,10),(0,28),(18,25),(1,24))$
$((23,15),(19,27),(16,22),(7,18),(9,29),(6,26),(10,28),(5,13),(11,2),(4,14)$,
$(25,1),(8,20),(0,12),(17,21),(24,3))$
$((10,27),(19,7),(23,4),(18,2),(11,5),(28,15),(25,16),(12,21),(9,24),(8,17)$,
$(0,22),(20,6),(29,1),(13,26),(3,14))$
$((15,5),(17,2),(13,21),(9,14),(1,19),(11,7),(10,24),(16,26),(22,8),(23,0)$,
$(25,6),(27,4),(29,18),(28,3),(12,20))$
$((29,6),(19,24),(23,7),(25,11),(5,26),(28,17),(3,21),(13,22),(15,8),(0,16)$,
$(2,12),(14,27),(9,20),(4,10),(1,18))$
$((8,28),(22,2),(14,5),(3,18),(10,6),(13,25),(19,26),(9,27),(0,11),(7,16)$,
$(17,23),(24,4),(21,1),(29,12),(20,15))$
$((19,25),(2,27),(13,4),(15,21),(12,7),(14,6),(11,20),(16,23),(0,24),(1,26)$,
$(22,17),(18,5),(9,28),(3,29),(10,8))$
$((26,18),(4,22),(12,28),(8,11),(25,15),(27,1),(21,7),(24,6),(0,29),(17,20)$,
$(14,23),(19,5),(9,10),(3,16),(13,2))$
$((15,9),(5,16),(17,26),(12,3),(8,24),(27,18),(13,28),(2,20),(0,21),(19,4)$,
$(6,23),(1,11),(14,25),(10,29),(7,22))$
$((0,17),(1,16),(2,28),(3,22),(4,25),(5,23),(6,12),(7,26),(8,19),(9,11)$,
$(10,21),(13,29),(14,20),(15,27),(18,24))$
$\alpha=6, \beta=8:$
$((6,26,7,21,12),(9,29,13,20,16),(18,4,11,1,27),(25,10,5,17,2),(3,23,14,28,15),(24,0,22,19,8))$ $((4,27,8,17,24),(1,19,0,23,18),(25,14,2,21,13),(20,5,26,12,9),(11,7,15,6,28),(10,3,22,16,29))$ $((19,24,14,26,4),(17,3,11,9,27),(1,25,5,18,22),(13,23,8,12,7),(21,15,20,10,0),(29,6,16,28,2))$ $((4,13,27,12,28),(17,20,14,5,21),(10,2,24,1,26),(16,3,18,29,0),(23,9,15,8,11),(25,6,22,7,19))$ $((24,13,22,5,15),(8,20,18,0,14),(28,1,21,9,17),(29,7,27,3,19),(6,23,12,4,10),(16,25,11,2,26))$ $((27,15,23,17,0),(2,12,29,8,16),(20,1,13,28,3),(9,10,21,18,26),(22,14,7,25,4),(24,6,19,5,11))$ $((4,29),(24,12),(26,0),(21,11),(13,6),(10,22),(25,17),(19,23),(28,9),(18,8)$,
$(5,27),(20,7),(15,2),(14,3),(16,1))$
$((19,9),(10,7),(4,21),(26,3),(18,2),(29,11),(23,16),(1,14),(28,8),(20,12)$,
$(17,22),(25,15),(24,5),(13,0),(27,6))$
$((18,28),(11,6),(27,2),(21,19),(25,3),(23,7),(16,4),(5,12),(17,1),(9,13)$,
$(0,20),(22,15),(14,29),(24,10),(8,26))$
$((2,20),(28,7),(3,21),(10,1),(14,9),(0,25),(12,22),(11,27),(15,29),(13,8)$,
$(5,16),(18,24),(6,17),(26,19),(4,23))$
$((29,5),(9,22),(24,16),(11,0),(4,17),(6,20),(10,27),(25,8),(28,19),(12,1)$,
$(14,21),(15,26),(13,3),(7,18),(23,2))$
$((26,13),(18,9),(21,16),(17,7),(20,11),(22,2),(10,8),(14,6),(15,4),(29,1)$,
$(23,5),(0,28),(19,27),(12,25),(3,24))$
$((27,14),(29,17),(22,8),(19,2),(4,20),(5,13),(24,9),(18,25),(6,21),(3,12)$,
$(23,1),(11,26),(0,15),(16,7),(10,28))$
$((0,12),(1,15),(2,13),(3,29),(4,14),(5,28),(6,18),(7,24),(8,21),(9,25)$,
$(10,23),(11,22),(16,27),(17,26),(19,20))$
$\alpha=7, \beta=6$ :
$((20,12,24,9,18),(5,21,19,29,16),(26,1,22,17,0),(3,27,8,11,25),(2,13,4,14,28),(7,23,15,6,10))$ $((7,15,29,6,13),(8,16,20,3,28),(17,21,14,24,1),(23,9,27,12,4),(22,19,26,11,5),(10,0,18,2,25))$
$((26,5,19,24,16),(9,22,13,1,15),(25,7,17,29,18),(11,4,27,14,0),(8,20,6,28,10),(23,2,21,3,12))$ $((24,0,21,8,13),(20,1,10,4,17),(12,26,3,18,28),(7,29,2,27,16),(22,15,25,6,11),(19,9,14,5,23))$ $((13,25,8,22,0),(5,15,3,17,24),(21,4,28,16,9),(12,7,26,10,2),(29,1,18,23,14),(20,19,6,27,11))$ $((4,29,12,1,19),(0,15,26,8,23),(14,20,2,11,7),(25,5,28,13,9),(16,22,18,24,3),(17,27,10,21,6))$ $((26,18,8,29,13),(4,15,21,16,25),(17,28,7,20,9),(12,0,19,27,5),(1,14,2,24,11),(23,6,22,10,3))$ $((11,23),(7,24),(14,25),(5,10),(13,20),(4,16),(0,27),(9,29),(6,18),(1,21)$,
$(8,17),(19,3),(22,12),(26,2),(28,15))$
$((3,11),(21,13),(18,27),(17,5),(0,29),(24,10),(23,16),(2,15),(28,9),(6,12)$,
$(8,19),(4,20),(22,7),(14,26),(25,1))$
$((29,5),(21,12),(17,26),(6,14),(24,4),(23,1),(8,15),(2,16),(20,0),(25,19)$,
$(3,22),(28,11),(7,18),(9,10),(27,13))$
$((13,23),(18,4),(28,0),(17,25),(14,3),(26,9),(27,15),(1,16),(6,24),(11,21)$,
$(5,20),(12,8),(7,19),(29,10),(22,2))$
$((9,12),(15,20),(29,11),(25,0),(28,19),(5,18),(21,7),(4,26),(1,27),(13,3)$,
$(16,6),(22,14),(17,2),(23,10),(24,8))$
$((0,16),(23,17),(15,24),(2,19),(4,22),(18,21),(5,13),(7,27),(8,14),(1,28)$,
$(26,6),(3,29),(25,12),(20,10),(11,9))$
$\alpha=8, \beta=4$ :
$((1,10,7,22,15),(13,8,11,20,4),(23,14,3,29,6),(17,25,5,16,24),(27,0,19,26,12),(18,2,21,9,28))$ $((19,7,27,13,29),(11,0,26,5,28),(21,18,23,8,10),(6,20,15,9,14),(16,4,17,22,2),(3,24,1,25,12))$ $((7,12,8,15,28),(5,21,1,13,24),(29,4,23,17,0),(3,27,19,22,16),(9,11,6,26,10),(25,2,14,20,18))$ $((11,26,16,21,4),(1,28,6,25,14),(0,24,10,23,13),(8,19,3,15,27),(18,29,5,17,7),(22,9,20,2,12))$ $((8,28,10,0,18),(29,11,22,6,12),(2,13,25,3,23),(20,16,9,19,1),(26,4,24,7,14),(21,17,27,5,15))$ $((9,23,19,4,27),(22,18,26,17,1),(25,10,29,2,15),(13,6,16,28,3),(20,8,24,11,7),(21,12,5,14,0))$ $((20,13,22,3,17),(18,24,14,28,4),(5,11,27,2,10),(12,1,16,23,0),(7,15,29,9,26),(8,21,6,19,25))$ $((6,10,3,18,27),(25,0,20,12,9),(19,28,17,2,24),(13,26,1,11,21),(14,8,16,7,29),(15,4,22,5,23))$ $((28,0),(14,4),(2,19),(24,12),(16,25),(23,11),(13,5),(22,8),(7,21),(1,29)$,
$(9,18),(27,10),(3,20),(17,6),(26,15))$
$((28,2),(21,3),(24,15),(11,25),(20,10),(0,16),(7,23),(8,26),(4,12),(17,29)$,
$(6,18),(14,22),(19,5),(13,9),(1,27))$
$((21,19),(16,29),(1,18),(6,15),(14,27),(23,12),(8,17),(20,5),(13,28),(26,2)$,
$(11,3),(9,24),(22,0),(10,4),(7,25))$
$((6,24),(26,3),(12,28),(15,0),(19,20),(8,29),(10,22),(9,17),(14,21),(27,16)$,
$(2,11),(1,23),(4,25),(18,5),(13,7))$
$\alpha=9, \beta=2$ :
$((20,7,15,9,10),(3,14,24,6,26),(21,0,29,5,11),(13,28,8,16,25),(23,19,27,1,12),(4,22,18,2,17))$ $((6,23,10,5,14),(20,11,29,8,15),(12,21,1,17,26),(2,24,18,3,13),(16,9,25,4,27),(22,7,28,19,0))$ $((5,25,6,20,12),(24,8,18,21,13),(27,17,9,23,2),(26,14,4,10,0),(22,11,1,28,15),(19,7,29,16,3))$ $((20,9,18,1,16),(5,26,2,11,23),(4,29,14,0,24),(13,7,12,8,27),(15,21,17,3,25),(10,28,6,19,22))$ $((7,27,11,28,14),(25,17,0,12,2),(22,9,21,8,13),(4,18,26,1,23),(16,6,15,5,24),(29,10,3,20,19))$ $((25,19,24,10,8),(20,14,23,7,17),(26,9,29,1,13),(22,12,6,21,5),(27,15,4,11,3),(2,16,0,18,28))$ $((14,9,19,5,27),(17,24,12,4,28),(10,6,13,0,25),(8,11,26,16,23),(7,21,2,20,18),(1,22,3,29,15))$ $((0,11,7,24,15),(1,10,26,4,20),(2,14,25,12,29),(3,23,13,9,28),(5,17,6,27,18),(8,19,21,16,22))$ $((0,20,13,5,28),(1,14,8,26,19),(2,15,23,17,22),(3,12,27,9,24),(4,16,7,10,21),(6,11,25,18,29))$ $((0,23),(1,24),(2,19),(3,21),(4,13),(5,16),(6,18),(7,25),(8,20),(9,11)$,
$(10,27),(12,28),(14,22),(15,26),(17,29))$
$((0,27),(1,25),(2,10),(3,15),(4,19),(5,20),(6,22),(7,26),(8,17),(9,12)$,
$(11,24),(13,29),(14,21),(16,28),(18,23))$
$\alpha=10, \beta=0:$
$((0,10,1,12,20),(2,14,5,11,21),(3,15,22,4,25),(6,13,24,7,26),(8,17,23,18,29),(9,16,28,19,27))$ $((1,11,2,13,21),(3,10,6,12,22),(4,16,23,0,26),(7,14,20,8,27),(9,18,24,19,25),(5,17,29,15,28))$
$((2,12,3,14,22),(4,11,7,13,23),(0,17,24,1,27),(8,10,21,9,28),(5,19,20,15,26),(6,18,25,16,29))$ $((3,13,4,10,23),(0,12,8,14,24),(1,18,20,2,28),(9,11,22,5,29),(6,15,21,16,27),(7,19,26,17,25))$ $((4,14,0,11,24),(1,13,9,10,20),(2,19,21,3,29),(5,12,23,6,25),(7,16,22,17,28),(8,15,27,18,26))$ $((0,13,20,4,28),(1,19,22,9,23),(2,18,3,17,27),(5,15,24,8,16),(6,11,25,12,21),(7,10,26,14,29))$ $((1,14,21,0,29),(2,15,23,5,24),(3,19,4,18,28),(6,16,20,9,17),(7,12,26,13,22),(8,11,27,10,25))$ $((2,10,22,1,25),(3,16,24,6,20),(4,15,0,19,29),(7,17,21,5,18),(8,13,27,14,23),(9,12,28,11,26))$ $((3,11,23,2,26),(4,17,20,7,21),(0,16,1,15,25),(8,18,22,6,19),(9,14,28,10,24),(5,13,29,12,27))$ $((4,12,24,3,27),(0,18,21,8,22),(1,17,2,16,26),(9,19,23,7,15),(5,10,29,11,20),(6,14,25,13,28))$
$K_{4(10)}$ Let the vertex set of $K_{4(10)}$ be $\mathbb{Z}_{40}$ with vertex partition $\{0,1, \ldots, 9\},\{10,11, \ldots, 19\},\{20, \ldots, 29\}$, and $\{30,31, \ldots, 39\}$. Since a 1 -factorization of $K_{4}$ and of $K_{10,10}$ exists, clearly, $K_{4(10)}=30 K_{2}$. Also, by [3], $K_{4(10)}=15 C_{5}$. For $1 \leq \alpha \leq 14$, the factorizations are:
$\alpha=1, \beta=28$ :
$((3,18,35,8,37),(1,17,34,6,22),(4,31,21,7,11),(10,33,23,15,39),(30,2,25,38,20)$, $(36,12,24,14,5),(13,29,0,16,28),(9,26,19,32,27))$
$((32,16),(12,3),(20,18),(28,9),(13,5),(23,4),(17,30),(0,33),(6,39),(14,31),(2,29),(26,8)$,
$(1,35),(38,7),(24,15),(34,11),(19,27),(37,21),(10,25),(22,36))$
$((28,14),(12,33),(35,16),(34,7),(10,31),(11,38),(23,18),(19,1),(4,15),(36,13),(24,8),(5,26)$, $(32,29),(6,30),(2,37),(22,3),(39,25),(0,27),(21,17),(20,9))$
$((23,5),(27,37),(17,29),(1,34),(21,30),(33,28),(6,12),(15,9),(35,10),(24,31),(19,20),(13,25)$,
$(0,11),(18,38),(22,4),(2,14),(36,16),(32,3),(26,7),(8,39))$
$((0,23),(38,1),(28,17),(33,24),(39,16),(11,22),(25,4),(37,13),(36,6),(21,35),(10,32),(34,18)$, $(8,19),(29,5),(12,9),(7,20),(15,30),(27,2),(14,3),(26,31))$
$((3,29),(35,4),(28,7),(0,12),(24,38),(20,14),(13,6),(5,18),(19,2),(39,22),(17,33),(11,1)$, $(25,32),(23,30),(9,21),(26,34),(16,27),(15,37),(8,31),(10,36))$
$((19,9),(24,0),(14,34),(18,37),(1,28),(20,35),(13,22),(7,31),(3,16),(27,12),(5,17),(30,8)$, $(25,11),(10,23),(33,2),(32,21),(36,29),(4,39),(15,6),(26,38))$
$((18,8),(13,7),(29,14),(10,21),(3,28),(33,5),(27,1),(36,2),(9,35),(26,30),(22,0),(19,37)$, $(38,16),(6,20),(34,4),(15,25),(31,23),(17,24),(39,12),(11,32))$
$((19,28),(8,33),(2,31),(10,29),(4,16),(15,0),(7,27),(5,37),(35,14),(34,20),(23,36),(18,22)$, $(24,32),(12,26),(39,17),(21,1),(25,6),(9,30),(13,38),(11,3))$
$((28,5),(27,10),(25,7),(33,15),(3,17),(35,0),(12,2),(32,26),(31,22),(8,11),(18,1),(34,21)$, $(9,37),(30,16),(13,23),(14,6),(38,4),(24,36),(29,19),(20,39))$
$((11,36),(15,31),(25,18),(6,16),(10,26),(28,30),(19,35),(27,17),(1,29),(32,0),(13,3),(21,33)$, $(24,34),(14,23),(2,39),(5,38),(12,8),(9,22),(4,20),(37,7))$
$((22,16),(13,9),(36,15),(31,6),(19,3),(5,32),(23,17),(33,29),(14,39),(0,21),(11,26),(38,27)$, $(10,37),(2,34),(20,1),(30,18),(12,28),(8,25),(4,24),(7,35))$
$((19,7),(8,10),(12,34),(20,36),(15,32),(4,33),(24,3),(21,6),(18,27),(30,25),(5,22),(1,31)$, $(17,37),(13,26),(39,9),(14,0),(35,28),(23,11),(29,38),(2,16))$
$((29,12),(22,37),(24,9),(31,27),(2,13),(36,17),(16,25),(38,8),(11,39),(1,14),(28,15),(20,5)$, $(21,4),(26,18),(7,33),(19,34),(30,3),(10,0),(32,6),(23,35))$
$((17,35),(32,28),(24,6),(39,3),(27,11),(22,7),(26,4),(18,36),(19,38),(2,15),(34,29),(13,30)$, $(5,10),(25,1),(37,23),(20,33),(21,12),(14,9),(16,8),(0,31))$
$((19,33),(5,16),(35,27),(18,28),(23,2),(13,34),(11,21),(4,36),(37,24),(17,9),(20,15),(3,38)$, $(0,30),(7,14),(22,8),(32,12),(6,26),(29,39),(1,10),(25,31))$
$((35,3),(32,8),(17,31),(13,4),(18,6),(14,26),(19,39),(20,0),(30,24),(1,12),(21,16),(33,27)$, $(11,2),(34,23),(9,29),(37,25),(38,22),(7,10),(5,15),(36,28))$
$((34,15),(24,13),(35,2),(22,30),(21,19),(8,14),(27,6),(12,23),(18,29),(33,1),(39,7),(38,28)$, $(32,20),(16,26),(31,5),(10,3),(0,17),(4,37),(11,9),(36,25))$
$((32,13),(29,35),(38,0),(36,8),(33,22),(20,2),(4,28),(6,37),(39,26),(10,30),(15,1),(27,3)$, $(18,24),(12,5),(11,31),(17,7),(23,19),(16,34),(14,21),(25,9))$
$((14,32),(21,15),(4,17),(1,16),(13,39),(0,19),(24,10),(18,9),(22,35),(27,8),(7,23),(30,12)$, $(20,31),(6,29),(34,5),(11,28),(25,33),(3,36),(2,38),(37,26))$
$((15,26),(36,9),(3,23),(8,17),(25,12),(29,4),(18,21),(35,24),(37,11),(22,34),(20,10),(14,38)$, $(39,0),(16,31),(33,6),(30,19),(5,27),(13,1),(2,28),(32,7))$
$((35,12),(13,21),(11,20),(2,24),(22,19),(29,8),(18,33),(16,37),(38,23),(15,3),(0,26),(36,7)$, $(6,17),(9,31),(25,14),(34,27),(28,39),(1,32),(10,4),(30,5))$
$((30,1),(10,6),(12,4),(28,0),(23,16),(33,13),(34,8),(31,29),(18,7),(36,14),(15,27),(32,22)$, $(39,5),(3,26),(20,37),(35,11),(2,21),(25,17),(9,38),(24,19))$
$((9,23),(2,17),(35,15),(26,1),(24,16),(21,3),(38,10),(8,13),(34,0),(28,31),(4,32),(29,37)$, $(39,18),(7,30),(6,19),(14,22),(27,36),(11,33),(12,20),(5,25))$
$((18,4),(38,15),(11,5),(25,3),(26,36),(34,10),(29,7),(21,8),(9,16),(31,19),(6,35),(33,14)$, $(2,32),(12,22),(24,39),(37,28),(23,1),(27,30),(0,13),(20,17))$ $((12,31),(29,11),(23,8),(4,19),(32,9),(35,5),(1,36),(27,13),(25,34),(6,38),(37,0),(17,26)$, $(14,30),(16,33),(3,20),(22,15),(28,10),(24,7),(39,21),(18,2))$ $((15,29),(34,28),(13,31),(14,4),(1,24),(36,19),(20,8),(3,33),(25,0),(9,10),(26,35),(37,12)$, $(27,39),(16,7),(30,11),(32,18),(5,21),(17,38),(2,22),(23,6))$ $((31,3),(37,14),(10,2),(25,19),(33,26),(35,13),(5,24),(23,32),(20,16),(22,17),(1,39),(38,12)$, $(30,29),(27,4),(34,9),(36,21),(15,7),(28,8),(0,18),(6,11))$
$((0,36),(1,37),(2,26),(3,34),(4,30),(5,19),(6,28),(7,12),(8,15),(9,33),(10,22),(11,24)$, $(13,20),(14,27),(16,29),(17,32),(18,31),(21,38),(23,39),(25,35))$
$\alpha=2, \beta=26:$
$((11,5,29,9,36),(35,25,12,6,13),(34,23,8,28,1),(27,39,20,19,0),(37,14,38,17,22)$,
$(7,21,4,26,18),(15,33,24,30,2),(32,10,31,3,16))$
$((12,24,10,29,32),(11,35,9,19,21),(28,4,15,37,3),(38,23,0,16,5),(31,1,39,17,20)$,
$(27,13,30,26,2),(18,22,36,6,25),(34,7,14,33,8))$
$((32,20),(13,24),(29,4),(34,9),(7,25),(30,28),(18,37),(6,38),(5,15),(3,10),(2,22),(21,0)$, $(17,33),(1,26),(14,23),(39,16),(27,19),(11,31),(35,8),(36,12))$ $((8,19),(37,11),(32,5),(12,20),(28,31),(10,21),(3,24),(36,7),(35,15),(23,2),(25,17),(9,39)$, $(13,1),(16,34),(0,22),(30,18),(14,6),(33,26),(4,27),(38,29))$
$((24,1),(9,13),(31,7),(8,30),(28,14),(25,0),(18,29),(17,37),(26,32),(39,2),(3,34),(27,11)$, $(4,36),(35,6),(12,38),(20,16),(5,10),(15,21),(22,33),(19,23))$
$((10,30),(20,7),(33,12),(25,15),(24,32),(35,28),(21,18),(23,39),(14,0),(17,5),(29,13),(3,26)$, $(22,1),(34,27),(31,19),(36,2),(8,11),(38,4),(6,16),(37,9))$
$((24,18),(3,36),(25,32),(19,29),(35,1),(27,5),(39,26),(38,2),(30,23),(28,16),(10,20),(37,12)$, $(13,8),(34,17),(6,22),(4,14),(0,31),(15,7),(21,33),(9,11))$ $((11,6),(29,15),(35,4),(8,26),(13,2),(32,14),(36,20),(18,28),(22,5),(21,31),(37,27),(23,7)$, $(9,24),(16,30),(10,33),(0,17),(25,34),(38,3),(39,19),(12,1))$ $((23,36),(35,24),(14,2),(7,32),(22,3),(0,38),(28,9),(13,26),(17,4),(20,11),(12,39),(31,5)$, $(10,34),(21,8),(30,19),(37,25),(16,27),(18,1),(33,29),(6,15))$
$((7,11),(32,2),(21,6),(13,20),(37,23),(31,4),(30,27),(24,16),(36,29),(17,26),(39,3),(19,1)$, $(15,28),(10,35),(22,34),(8,12),(5,18),(38,25),(33,0),(14,9))$ $((21,12),(22,10),(27,14),(9,25),(29,16),(33,1),(19,32),(8,36),(18,35),(7,28),(2,24),(11,0)$, $(39,4),(5,37),(6,23),(20,3),(30,17),(34,26),(15,31),(38,13))$ $((22,19),(8,25),(0,10),(9,38),(3,18),(29,12),(37,21),(5,24),(15,36),(34,13),(2,17),(27,32)$, $(35,26),(28,6),(16,7),(30,20),(33,4),(39,11),(31,14),(23,1))$
$((12,0),(32,4),(34,6),(8,10),(26,5),(24,11),(29,3),(13,37),(16,23),(31,9),(19,33),(7,39)$, $(18,2),(17,27),(38,28),(30,25),(14,20),(15,22),(21,35),(36,1))$ $((2,19),(17,9),(10,7),(21,30),(33,3),(26,15),(29,35),(25,36),(31,12),(16,4),(13,32),(14,1)$, $(22,38),(37,0),(11,34),(8,24),(20,5),(27,6),(18,23),(39,28))$
$((19,34),(29,8),(7,13),(1,25),(2,11),(0,24),(14,26),(5,30),(32,17),(23,3),(4,18),(39,10)$, $(6,33),(38,15),(22,35),(37,20),(28,12),(9,27),(21,36),(16,31))$ $((39,21),(19,3),(10,23),(29,34),(30,11),(15,0),(16,22),(27,12),(7,24),(37,28),(32,6),(25,4)$, $(36,26),(31,13),(20,8),(1,38),(17,35),(18,9),(2,33),(14,5))$
$((30,1),(39,0),(11,28),(33,27),(32,22),(23,12),(24,4),(3,13),(9,26),(18,34),(10,37),(35,16)$, $(7,19),(15,8),(29,31),(5,36),(2,21),(6,17),(14,25),(38,20))$
$((24,15),(39,29),(13,5),(12,35),(25,11),(9,30),(34,2),(31,18),(38,16),(19,28),(22,8),(0,20)$, $(14,3),(6,10),(21,32),(37,4),(1,27),(26,7),(23,33),(17,36))$
$((37,16),(15,23),(22,11),(9,32),(13,0),(5,19),(12,7),(25,39),(28,34),(2,29),(24,38),(36,18)$, $(3,35),(8,27),(14,21),(10,4),(30,6),(1,17),(33,20),(26,31))$
$((4,23),(39,22),(11,38),(28,17),(14,24),(32,18),(16,21),(25,13),(20,2),(30,29),(9,33),(7,35)$, $(15,1),(6,31),(19,36),(0,34),(3,27),(5,12),(10,26),(37,8))$
$((23,35),(32,1),(25,2),(18,6),(33,13),(26,37),(3,21),(12,4),(27,15),(20,9),(7,22),(5,34)$, $(24,39),(29,11),(0,30),(17,31),(36,16),(38,19),(28,10),(8,14))$
$((36,24),(19,6),(9,15),(17,7),(2,35),(38,21),(14,34),(30,4),(1,20),(29,37),(3,12),(10,27)$, $(31,22),(5,23),(33,18),(32,0),(11,26),(13,28),(8,39),(16,25))$
$((28,33),(34,24),(38,18),(23,17),(12,22),(13,4),(0,29),(3,32),(39,14),(26,6),(35,5),(25,31)$, $(27,36),(30,7),(19,37),(16,8),(10,2),(20,15),(1,11),(21,9))$
$((22,13),(24,17),(8,18),(20,6),(35,27),(30,15),(33,25),(0,36),(37,7),(39,5),(10,1),(19,4)$, $(11,3),(16,2),(32,28),(29,14),(31,23),(12,9),(34,21),(38,26))$
$((25,10),(6,39),(35,14),(12,34),(28,36),(24,19),(15,32),(5,21),(0,18),(38,27),(13,23),(2,31)$, $(20,4),(37,1),(29,7),(33,11),(9,22),(16,26),(8,17),(3,30))$
$((37,24),(20,35),(13,39),(38,10),(14,36),(27,18),(17,3),(0,28),(30,22),(9,16),(34,15),(12,2)$, $(4,11),(29,6),(7,33),(23,32),(8,31),(1,21),(25,5),(19,26))$ $((1,29),(36,13),(11,23),(12,30),(24,6),(15,39),(7,38),(33,16),(19,35),(26,0),(27,31),(3,25)$, $(22,14),(18,20),(17,21),(34,4),(37,2),(10,9),(32,8),(5,28))$ $((0,35),(1,16),(2,28),(3,15),(4,22),(5,33),(6,37),(7,27),(8,38),(9,23),(10,36),(11,32)$, $(12,26),(13,21),(14,30),(17,29),(18,39),(19,25),(20,34),(24,31))$
$\alpha=3, \beta=24$ :
$((13,9,28,30,26),(37,19,35,27,4),(10,1,33,17,0),(8,11,3,15,23),(5,24,2,21,32)$,
$(22,16,7,34,6),(36,25,39,14,29),(31,12,38,20,18))$
$((29,5,30,19,7),(16,3,14,34,20),(12,24,18,21,37),(2,17,38,15,28),(33,25,1,39,10)$,
$(8,22,11,6,13),(27,32,26,36,0),(35,9,31,23,4))$
$((34,24,39,2,27),(11,28,35,23,7),(12,33,21,31,0),(4,10,5,20,36),(38,25,16,32,29)$, $(1,15,8,14,37),(13,22,3,18,30),(6,17,26,9,19))$ $((24,32),(1,21),(4,13),(0,35),(23,12),(16,30),(29,37),(18,8),(14,33),(11,9),(10,20),(26,2)$, $(17,3),(39,19),(34,28),(6,15),(36,22),(27,7),(25,31),(38,5))$
$((32,1),(0,24),(11,38),(31,3),(17,9),(27,19),(30,23),(10,21),(8,20),(18,5),(22,34),(28,13)$, $(29,4),(33,26),(15,7),(12,35),(25,37),(6,39),(16,2),(14,36))$ $((5,34),(12,28),(13,25),(2,37),(27,30),(7,32),(35,26),(10,6),(0,33),(14,22),(11,31),(38,23)$, $(20,15),(9,36),(19,4),(18,29),(8,24),(16,1),(17,21),(3,39))$
$((34,9),(11,32),(31,6),(14,5),(30,17),(10,35),(21,15),(39,0),(37,24),(18,28),(16,23),(12,8)$, $(4,26),(3,29),(27,36),(19,20),(25,2),(33,7),(38,13),(1,22))$
$((35,15),(6,38),(18,37),(7,24),(21,16),(8,17),(3,30),(9,27),(12,4),(10,22),(29,0),(14,1)$, $(13,36),(26,5),(23,33),(28,39),(11,34),(25,32),(2,19),(31,20))$
$((26,18),(39,11),(30,25),(8,34),(20,14),(0,32),(22,38),(19,1),(5,15),(31,4),(9,10),(24,17)$, $(28,16),(21,36),(2,29),(3,35),(23,37),(6,33),(7,12),(27,13))$ $((33,19),(29,35),(25,0),(28,7),(37,6),(38,27),(5,17),(15,34),(16,31),(8,30),(14,9),(36,11)$, $(3,20),(2,12),(22,18),(21,4),(39,23),(24,1),(26,10),(13,32))$ $((1,13),(28,3),(38,19),(8,36),(27,5),(10,23),(30,21),(33,16),(9,37),(32,2),(7,25),(11,26)$, $(14,6),(12,20),(35,22),(4,24),(15,39),(29,34),(18,0),(31,17))$ $((13,3),(33,5),(21,38),(20,39),(30,22),(8,32),(28,10),(7,36),(16,37),(31,2),(15,25),(29,11)$, $(18,27),(34,0),(17,35),(23,9),(1,26),(24,19),(14,4),(6,12))$
$((39,12),(6,16),(31,22),(35,11),(7,14),(32,10),(23,2),(0,19),(33,9),(37,5),(3,25),(38,18)$, $(26,34),(27,17),(8,28),(1,20),(13,21),(36,24),(15,4),(29,30))$
$((2,35),(9,38),(19,29),(36,12),(13,31),(11,20),(1,28),(37,3),(8,27),(15,26),(24,16),(33,18)$, $(39,5),(22,7),(30,10),(21,6),(32,23),(25,17),(0,14),(34,4))$
$((1,31),(29,16),(35,18),(30,11),(9,12),(4,32),(34,13),(15,27),(36,17),(6,28),(10,37),(19,3)$, $(25,5),(38,24),(23,0),(7,21),(33,20),(39,8),(14,26),(2,22))$
$((27,14),(0,28),(37,11),(8,10),(4,17),(19,25),(15,31),(36,23),(39,21),(16,35),(32,22),(30,1)$, $(2,18),(26,6),(3,34),(7,38),(20,9),(33,24),(13,29),(12,5))$
$((28,31),(4,39),(6,20),(37,15),(36,16),(24,10),(5,11),(3,38),(32,9),(14,2),(13,35),(23,19)$, $(0,26),(34,21),(22,12),(27,33),(17,29),(25,8),(30,7),(1,18))$
$((3,33),(13,20),(25,6),(9,22),(18,36),(26,39),(31,19),(7,35),(34,10),(12,30),(27,16),(17,37)$, $(4,38),(0,21),(32,14),(24,11),(15,2),(29,8),(23,1),(28,5))$
$((15,0),(11,25),(16,8),(14,21),(20,30),(4,22),(19,5),(6,24),(7,31),(28,33),(12,34),(2,13)$, $(39,27),(32,18),(3,36),(1,35),(37,26),(17,23),(29,9),(38,10))$
$((12,1),(25,18),(6,23),(10,31),(2,36),(33,11),(24,30),(8,19),(3,32),(15,29),(26,38),(7,13)$, $(27,37),(14,28),(16,0),(35,21),(39,9),(20,4),(5,22),(17,34))$
$((9,25),(6,36),(4,28),(29,33),(12,3),(34,18),(5,13),(19,26),(15,30),(35,14),(24,31),(11,23)$, $(22,37),(1,27),(17,32),(0,20),(39,16),(21,8),(2,38),(10,7))$
$((13,33),(21,3),(39,17),(27,31),(25,12),(32,28),(9,18),(8,38),(1,36),(11,0),(30,4),(10,29)$, $(5,16),(37,20),(19,22),(26,7),(14,23),(35,6),(24,15),(34,2))$ $((8,35),(28,37),(9,21),(39,22),(20,17),(10,2),(0,38),(1,29),(33,15),(30,6),(4,11),(26,31)$, $(36,19),(32,12),(34,16),(13,24),(23,5),(18,7),(14,25),(27,3))$
$((15,22),(23,18),(31,8),(33,4),(7,17),(2,30),(34,19),(5,21),(0,37),(14,24),(36,10),(38,28)$, $(39,13),(9,16),(29,6),(35,25),(26,3),(27,12),(11,1),(20,32))$
$((10,27),(21,11),(22,17),(14,30),(6,32),(20,2),(9,24),(29,31),(26,12),(18,39),(28,19),(13,0)$, $(38,16),(37,7),(23,3),(35,5),(34,1),(36,15),(4,25),(8,33))$ $((0,22),(37,13),(15,32),(6,18),(19,21),(11,27),(9,30),(39,7),(17,1),(2,33),(35,20),(16,4)$, $(26,8),(23,34),(36,28),(38,14),(3,24),(25,10),(12,29),(31,5))$ $((0,30),(1,38),(2,11),(3,10),(4,18),(5,36),(6,27),(7,20),(8,37),(9,15),(12,21),(13,23)$, $(14,31),(16,26),(17,28),(19,32),(22,33),(24,35),(25,34),(29,39))$
$\alpha=4, \beta=22$ :
$((37,18,9,38,7),(19,25,32,16,1),(36,17,27,10,8),(34,4,15,20,0),(21,31,23,3,39)$,
$(13,24,12,2,35),(6,33,22,30,29),(5,11,26,14,28))$
$((36,23,2,14,6),(11,25,4,10,22),(17,5,16,21,1),(13,32,24,8,38),(35,3,27,34,7)$,
$(9,29,0,26,37),(31,20,18,30,15),(33,12,28,39,19))$
$((2,13,0,23,34),(16,35,28,10,9),(15,26,36,1,27),(8,19,20,30,25),(32,21,6,39,17)$,
$(4,33,3,31,14),(7,11,37,24,18),(12,5,29,38,22))$
$((12,23,6,15,35),(32,27,37,19,7),(26,18,5,21,2),(8,34,20,33,29),(3,30,17,28,36)$,
$(10,24,11,1,31),(14,39,25,13,9),(0,38,16,4,22))$
$((34,11),(36,7),(35,19),(21,15),(22,13),(20,4),(31,8),(24,30),(23,38),(17,9),(14,37),(2,39)$,
$(12,6),(18,25),(10,29),(3,28),(0,16),(27,5),(1,33),(26,32))$
$((19,6),(20,13),(15,22),(33,24),(5,31),(1,39),(35,26),(32,18),(7,17),(25,16),(10,36),(34,29)$, $(27,9),(8,23),(37,21),(12,30),(0,14),(4,11),(3,38),(28,2))$
$((8,16),(18,28),(33,15),(25,34),(35,23),(2,10),(6,32),(36,19),(26,5),(12,1),(29,14),(11,0)$, $(37,4),(20,17),(24,9),(30,27),(22,39),(31,7),(3,13),(38,21))$ $((7,22),(10,1),(18,23),(5,25),(26,38),(34,19),(37,6),(28,13),(12,8),(15,2),(29,16),(21,33)$, $(39,20),(17,31),(27,35),(14,3),(32,11),(4,36),(24,0),(30,9))$ $((12,29),(22,32),(18,31),(17,6),(7,30),(13,26),(24,1),(36,0),(10,35),(11,28),(3,37),(25,15)$, $(8,27),(5,39),(2,20),(14,21),(38,19),(16,33),(23,4),(34,9))$ $((38,14),(30,28),(24,6),(3,17),(36,9),(32,4),(31,22),(0,27),(2,18),(11,20),(16,7),(12,39)$, $(25,10),(34,5),(21,19),(1,26),(8,35),(13,29),(23,33),(15,37))$ $((37,1),(9,28),(19,22),(18,3),(2,27),(30,8),(39,10),(17,0),(31,11),(25,14),(12,26),(34,24)$, $(7,33),(13,36),(38,20),(29,35),(23,32),(16,6),(21,4),(5,15))$ $((18,0),(35,21),(31,24),(38,5),(33,27),(36,11),(7,28),(3,10),(34,14),(23,39),(26,6),(12,37)$, $(17,25),(30,19),(32,8),(22,1),(16,2),(29,15),(9,20),(4,13))$ $((32,2),(7,23),(1,30),(25,3),(39,9),(15,38),(36,20),(22,8),(27,16),(13,33),(4,31),(12,34)$, $(18,35),(26,10),(19,5),(24,14),(0,37),(6,28),(11,21),(29,17))$
$((36,16),(30,4),(18,29),(27,31),(21,9),(14,33),(13,1),(11,23),(7,20),(32,10),(15,24),(2,38)$, $(8,39),(0,12),(5,37),(35,17),(25,6),(28,34),(19,26),(22,3))$ $((6,10),(5,32),(9,19),(16,3),(12,38),(34,13),(37,23),(17,2),(26,30),(20,8),(24,36),(29,11)$, $(18,21),(28,31),(1,25),(0,33),(15,7),(4,35),(22,14),(27,39))$ $((32,28),(22,17),(25,7),(29,36),(15,3),(4,18),(27,38),(5,24),(20,1),(19,23),(26,39),(11,33)$, $(14,35),(2,37),(30,16),(21,10),(34,6),(0,31),(13,8),(9,12))$
$((5,13),(20,14),(19,29),(37,28),(38,18),(27,7),(10,33),(26,9),(30,21),(2,24),(23,17),(11,6)$, $(35,25),(12,3),(1,34),(36,22),(15,8),(31,16),(39,4),(32,0))$
$((25,9),(27,13),(32,19),(16,20),(21,12),(39,7),(22,2),(38,10),(30,0),(36,15),(35,11),(26,33)$, $(3,34),(18,8),(5,23),(28,4),(1,14),(37,29),(17,24),(31,6))$
$((9,23),(30,10),(14,5),(3,29),(34,26),(31,25),(36,27),(18,6),(24,38),(32,15),(8,11),(28,1)$, $(4,17),(21,13),(33,2),(37,20),(19,0),(22,35),(39,16),(12,7))$ $((5,35),(21,36),(33,8),(24,19),(39,18),(25,12),(29,1),(31,9),(37,10),(7,13),(38,6),(22,16)$, $(26,3),(20,32),(23,15),(34,17),(28,0),(30,14),(11,2),(4,27))$ $((10,20),(36,2),(37,22),(9,33),(12,27),(17,8),(32,29),(1,18),(28,16),(5,30),(13,23),(4,26)$, $(14,7),(39,24),(0,15),(25,38),(19,31),(34,21),(35,6),(11,3))$ $((16,34),(17,26),(3,21),(14,36),(38,4),(30,11),(1,15),(37,8),(2,29),(6,20),(9,32),(23,10)$, $(12,31),(5,22),(27,18),(0,35),(39,13),(19,28),(25,33),(7,24))$
$((37,25),(12,20),(26,16),(21,8),(13,31),(27,14),(17,33),(32,3),(35,24),(15,28),(6,22),(9,11)$, $(19,2),(0,39),(18,34),(30,23),(36,5),(7,10),(1,38),(4,29))$ $((3,24),(35,9),(29,39),(32,12),(21,7),(31,26),(11,27),(8,14),(13,6),(4,19),(38,17),(23,1)$, $(33,28),(0,10),(36,25),(5,20),(2,30),(16,37),(15,34),(18,22))$ $((37,17),(14,23),(10,34),(19,3),(7,26),(31,29),(33,5),(30,13),(25,2),(9,22),(12,4),(0,21)$, $(20,35),(15,39),(28,8),(27,6),(1,32),(18,36),(16,24),(11,38))$ $((0,25),(1,35),(2,31),(3,20),(4,24),(5,10),(6,30),(7,29),(8,26),(9,15),(11,39),(12,36)$, $(13,37),(14,32),(16,23),(17,21),(18,33),(19,27),(22,34),(28,38))$
$\alpha=5, \beta=20$ :
$((24,1,34,11,38),(7,12,36,23,16),(30,26,37,21,19),(32,15,28,2,17),(25,0,33,13,8)$,
$(4,29,35,27,10),(6,14,39,3,20),(31,18,5,22,9))$
$((16,8,29,12,4),(2,13,25,37,14),(9,34,22,1,26),(0,18,27,32,11),(36,6,24,31,17)$,
$(39,7,28,33,5),(3,19,38,15,21),(20,35,23,10,30))$
$((12,28,13,26,6),(9,30,21,2,33),(39,19,22,35,8),(27,1,31,29,11),(16,37,18,7,34)$,
$(24,3,36,25,5),(14,20,32,10,38),(4,15,23,0,17))$
$((32,0,34,14,22),(37,12,3,30,6),(9,29,2,26,39),(19,8,15,31,4),(20,11,5,35,10)$,
$(28,16,24,17,1),(25,18,36,27,33),(23,38,13,21,7))$
$((19,0,39,22,37),(15,24,4,13,1),(25,38,18,34,10),(14,23,31,11,33),(9,27,5,21,12)$, $(16,29,6,35,3),(32,2,36,8,26),(7,20,17,28,30))$
$((38,3),(0,20),(7,35),(2,15),(31,14),(30,25),(27,8),(23,12),(5,26),(19,1),(34,24),(21,10)$, $(4,39),(22,13),(11,28),(33,18),(16,9),(6,32),(37,17),(29,36))$
$((18,28),(24,19),(1,14),(20,15),(4,22),(37,23),(9,36),(3,31),(11,30),(27,12),(0,10),(16,35)$, $(17,29),(34,25),(6,13),(5,32),(39,2),(38,26),(33,7),(8,21))$ $((36,15),(24,35),(25,12),(31,27),(28,34),(5,38),(14,8),(22,11),(33,29),(37,13),(19,23),(30,4)$, $(6,21),(18,2),(20,1),(3,17),(32,9),(26,0),(39,16),(7,10))$ $((21,32),(29,3),(18,4),(13,31),(9,23),(35,25),(7,26),(2,38),(6,11),(5,34),(37,0),(12,1)$, $(8,20),(19,27),(22,15),(33,17),(36,14),(28,10),(16,30),(39,24))$ $((25,11),(20,9),(1,18),(31,26),(4,23),(30,17),(39,27),(6,34),(10,8),(22,2),(0,12),(19,29)$, $(3,32),(24,14),(13,5),(15,37),(28,35),(21,33),(7,38),(36,16))$ $((23,2),(18,26),(1,35),(25,14),(10,33),(27,34),(20,12),(39,13),(36,5),(28,37),(38,17),(30,8)$, $(0,29),(31,21),(3,22),(32,24),(4,11),(6,16),(15,7),(9,19))$
$((31,7),(20,19),(32,13),(3,34),(16,27),(17,21),(12,39),(38,9),(29,37),(10,2),(0,22),(24,33)$, $(4,35),(15,26),(18,8),(23,1),(5,30),(28,14),(36,11),(25,6))$
$((4,38),(29,15),(5,28),(2,16),(23,17),(35,9),(0,36),(14,27),(13,30),(12,8),(6,39),(33,20)$, $(32,25),(19,31),(24,10),(18,21),(3,37),(26,34),(22,7),(11,1))$
$((37,10),(35,18),(4,34),(0,24),(11,23),(1,16),(15,39),(30,2),(25,19),(9,21),(29,32),(22,17)$, $(5,20),(26,3),(36,28),(14,7),(38,12),(31,8),(6,33),(13,27))$
$((36,7),(27,6),(20,4),(23,3),(26,12),(37,24),(14,35),(16,21),(1,33),(15,5),(25,9),(10,31)$, $(19,32),(11,39),(34,2),(17,8),(30,29),(13,0),(18,22),(38,28))$
$((27,15),(8,23),(31,0),(6,22),(35,26),(10,5),(37,9),(20,39),(33,12),(29,38),(11,2),(13,36)$, $(30,1),(28,19),(7,17),(34,21),(3,25),(14,4),(16,32),(18,24))$ $((15,3),(6,19),(14,5),(2,20),(26,36),(32,28),(25,16),(10,22),(39,23),(38,0),(21,35),(33,8)$, $(9,17),(1,29),(27,7),(13,34),(37,4),(31,12),(18,30),(11,24))$ $((28,39),(3,13),(37,5),(22,16),(1,38),(8,34),(19,26),(25,15),(29,14),(27,0),(20,31),(21,36)$, $(24,30),(12,2),(10,6),(9,18),(17,35),(4,33),(11,7),(32,23))$ $((14,3),(39,21),(12,5),(17,6),(26,16),(30,23),(18,32),(27,2),(36,24),(8,11),(22,31),(33,19)$, $(4,25),(20,38),(1,37),(9,28),(29,10),(13,7),(0,35),(15,34))$ $((20,13),(29,5),(28,3),(17,39),(4,36),(8,38),(15,6),(32,1),(30,0),(21,11),(33,16),(18,23)$, $(19,34),(31,25),(24,7),(10,9),(27,37),(26,14),(12,22),(35,2))$
$((3,33),(27,30),(7,19),(16,5),(36,20),(9,24),(15,35),(39,25),(26,17),(22,8),(14,32),(11,37)$, $(4,28),(12,34),(0,21),(38,6),(13,23),(1,10),(18,29),(2,31))$
$((21,1),(2,24),(36,19),(15,30),(8,37),(9,14),(0,28),(33,22),(35,12),(18,39),(11,3),(25,7)$, $(5,23),(27,17),(20,34),(16,38),(26,10),(13,29),(31,6),(4,32))$
$((20,16),(9,13),(7,37),(3,18),(14,0),(28,31),(19,35),(39,10),(15,33),(26,11),(25,2),(38,27)$, $(5,17),(34,29),(36,1),(24,8),(30,22),(23,6),(12,32),(21,4))$ $((14,21),(7,32),(30,12),(25,1),(22,38),(2,19),(5,31),(13,24),(0,16),(15,9),(20,37),(3,27)$, $(10,36),(4,26),(34,17),(6,18),(29,39),(23,33),(11,35),(8,28))$ $((0,15),(1,39),(2,37),(3,10),(4,27),(5,19),(6,28),(7,29),(8,32),(9,11),(12,24),(13,35)$, $(14,30),(16,31),(17,25),(18,20),(21,38),(22,36),(23,34),(26,33))$
$\alpha=6, \beta=18:$
$((3,21,33,0,20),(9,36,27,12,22),(24,8,25,2,11),(39,4,10,5,29),(37,17,38,14,26)$,
$(15,31,23,7,35),(28,32,13,34,19),(30,6,16,1,18))$
$((34,25,9,21,2),(19,5,31,28,33),(1,35,18,8,27),(26,15,4,30,17),(0,29,12,6,11)$,
$(13,22,14,32,20),(16,24,36,3,38),(23,37,7,39,10))$
$((6,27,7,25,32),(12,31,4,34,8),(16,22,0,38,23),(17,24,3,33,9),(29,35,28,14,1)$,
$(11,5,13,2,20),(10,37,21,19,30),(18,36,15,39,26))$
$((37,2,15,8,20),(31,9,30,22,7),(23,13,3,14,0),(21,17,34,18,5),(12,4,25,39,1)$,
$(36,11,28,16,26),(29,32,27,38,19),(24,33,6,10,35))$
$((8,16,4,37,13),(27,34,6,23,15),(39,18,9,38,12),(29,33,17,1,36),(28,2,10,20,5)$,
$(21,11,35,22,31),(24,0,30,26,19),(32,3,25,14,7))$
$((23,12,21,4,19),(22,18,28,34,1),(30,11,25,16,29),(37,5,24,15,9),(14,8,26,32,2)$, $(6,31,3,39,20),(27,10,38,7,33),(0,13,35,17,36))$
$((12,0),(25,31),(28,15),(3,18),(14,24),(34,20),(22,39),(9,10),(5,30),(4,35),(32,1),(36,23)$,
$(6,37),(11,26),(13,21),(7,29),(38,2),(16,33),(17,8),(19,27))$
$((5,38),(26,12),(6,28),(20,18),(1,13),(21,8),(35,0),(9,24),(31,2),(17,32),(11,4),(33,23)$,
$(36,22),(14,39),(27,30),(34,15),(7,16),(37,3),(29,10),(25,19))$
$((16,34),(4,13),(27,2),(28,1),(25,37),(15,7),(31,29),(22,5),(3,26),(9,23),(14,36),(24,39)$, $(35,8),(21,38),(19,32),(17,20),(12,30),(33,11),(10,0),(6,18))$
$((0,32),(37,22),(28,7),(13,36),(33,26),(8,11),(35,14),(30,15),(23,4),(34,24),(20,9),(2,12)$, $(21,18),(38,6),(10,3),(25,5),(17,29),(19,39),(1,31),(16,27))$ $((12,3),(13,28),(21,35),(30,23),(9,39),(33,14),(0,17),(1,10),(4,29),(38,26),(19,20),(6,24)$, $(37,8),(5,16),(7,11),(36,2),(31,27),(32,15),(25,18),(22,34))$ $((17,5),(16,31),(19,9),(37,24),(33,1),(39,13),(14,4),(20,15),(3,30),(28,0),(23,11),(27,18)$, $(32,8),(22,2),(26,35),(21,6),(10,25),(12,34),(36,7),(38,29))$
$((16,20),(28,10),(22,8),(25,33),(18,7),(13,6),(14,34),(4,24),(3,23),(29,9),(0,39),(5,26)$, $(19,2),(27,35),(31,17),(32,21),(15,38),(30,1),(11,37),(12,36))$ $((4,20),(11,9),(28,17),(36,8),(6,29),(15,21),(19,31),(25,12),(33,18),(32,23),(34,3),(2,30)$, $(5,39),(35,16),(27,0),(24,7),(14,37),(22,10),(26,13),(38,1))$
$((33,22),(5,27),(31,13),(16,39),(38,28),(26,2),(19,7),(32,24),(10,8),(4,17),(0,34),(20,35)$, $(29,37),(12,9),(3,15),(14,6),(25,36),(21,30),(1,11),(23,18))$
$((12,33),(27,37),(11,39),(2,17),(23,35),(4,38),(25,6),(29,34),(24,13),(32,22),(14,20),(5,36)$, $(8,31),(18,0),(9,16),(10,26),(15,1),(28,30),(21,7),(19,3))$ $((0,26),(27,13),(1,24),(8,19),(35,25),(17,6),(9,14),(10,36),(22,3),(29,2),(7,12),(20,30)$, $(11,31),(38,18),(37,15),(28,39),(23,34),(21,16),(4,32),(33,5))$ $((24,2),(38,22),(34,21),(29,3),(19,37),(7,13),(31,18),(4,36),(25,0),(27,11),(33,8),(15,5)$, $(30,16),(35,12),(28,9),(20,1),(32,10),(17,39),(14,23),(6,26))$
$((39,23),(2,18),(17,27),(29,8),(13,38),(1,37),(19,36),(26,9),(33,10),(31,14),(24,30),(22,11)$, $(28,4),(12,32),(15,25),(0,21),(7,20),(35,6),(34,5),(16,3))$
$((22,19),(26,7),(14,21),(11,34),(18,24),(36,28),(25,30),(27,4),(8,39),(3,17),(31,10),(9,35)$, $(2,33),(1,23),(0,16),(38,20),(5,32),(15,6),(29,13),(12,37))$
$((35,3),(36,20),(8,28),(25,1),(32,16),(37,0),(10,21),(33,15),(29,18),(30,13),(9,34),(17,7)$, $(6,19),(38,11),(14,5),(2,23),(24,12),(39,27),(31,26),(22,4))$
$((39,21),(5,23),(27,9),(20,12),(2,16),(11,3),(30,7),(28,37),(0,15),(36,6),(22,17),(24,31)$, $(4,33),(1,26),(10,34),(14,29),(8,38),(19,35),(25,13),(32,18))$
$((16,36),(19,0),(37,18),(38,25),(20,31),(11,29),(3,28),(12,5),(23,17),(32,9),(33,13),(22,15)$, $(4,26),(6,39),(24,10),(2,35),(14,27),(7,34),(1,21),(8,30))$ $((0,31),(1,19),(2,39),(3,27),(4,18),(5,35),(6,22),(7,10),(8,23),(9,13),(11,32),(12,28)$, $(14,30),(15,29),(16,37),(17,25),(20,33),(21,36),(24,38),(26,34))$
$\alpha=7, \beta=16$ :
$((12,39,20,2,33),(3,25,32,16,27),(14,5,10,1,29),(36,24,31,28,9),(19,35,7,34,22)$,
$(13,4,11,23,30),(18,6,17,8,26),(21,15,38,0,37))$
$((36,4,16,37,6),(26,32,20,31,17),(27,1,11,5,30),(12,23,18,34,2),(28,33,24,14,3)$,
$(19,39,0,22,7),(8,15,29,35,21),(10,9,13,25,38))$
$((36,25,31,14,7),(39,8,37,20,13),(38,23,4,26,9),(27,15,30,11,35),(21,32,2,19,3)$,
$(18,24,1,12,5),(0,16,28,6,10),(22,17,34,29,33))$
$((27,6,35,8,10),(32,24,30,28,19),(16,1,26,3,33),(25,18,9,39,4),(11,38,14,21,0)$,
$(20,7,23,34,12),(5,29,17,37,13),(22,31,15,36,2))$
$((27,0,12,9,32),(20,19,33,26,36),(38,17,23,3,13),(15,25,11,28,5),(10,2,16,31,29)$,
$(7,21,30,4,37),(1,34,8,18,22),(24,35,14,39,6))$
$((12,31,2,21,38),(33,18,20,5,23),(36,1,28,39,17),(7,24,34,3,15),(26,11,9,25,19)$, $(4,10,22,37,27),(16,8,14,0,35),(6,13,32,29,30))$ $((8,31,1,32,22),(15,6,38,3,20),(16,5,27,11,34),(0,17,7,18,30),(35,23,2,14,4)$,
$(19,24,13,36,21),(9,29,12,25,33),(37,26,39,10,28))$
$((25,34),(29,8),(15,4),(19,9),(39,1),(0,33),(12,24),(10,30),(6,11),(3,36),(35,22),(28,2)$,
$(23,13),(32,5),(26,7),(38,16),(21,31),(18,37),(27,17),(14,20))$
$((6,32),(26,10),(7,13),(3,30),(17,2),(14,33),(29,39),(22,36),(38,4),(24,37),(18,1),(28,15)$, $(25,16),(5,19),(34,9),(27,8),(21,11),(31,23),(12,35),(0,20))$
$((13,1),(17,9),(24,8),(34,14),(31,18),(39,7),(26,5),(36,19),(0,29),(37,25),(28,38),(23,6)$,
$(16,21),(22,11),(3,32),(20,4),(30,12),(35,15),(27,2),(33,10))$
$((22,30),(23,32),(19,27),(13,8),(26,12),(9,37),(10,35),(31,4),(25,39),(11,3),(0,28),(29,36)$, $(17,5),(15,34),(21,1),(14,6),(18,38),(33,20),(7,16),(24,2))$
$((6,22),(18,28),(4,19),(24,3),(39,2),(37,15),(23,10),(35,20),(1,17),(8,36),(30,9),(14,26)$, $(32,11),(21,33),(12,27),(16,29),(5,25),(34,13),(0,31),(7,38))$ $((16,9),(22,4),(31,11),(14,1),(34,5),(2,26),(19,8),(12,7),(17,20),(32,15),(36,18),(37,23)$, $(13,29),(27,38),(21,39),(28,35),(6,33),(25,30),(10,3),(24,0))$ $((34,28),(32,18),(6,29),(15,1),(23,39),(33,11),(13,0),(7,25),(5,37),(12,36),(4,17),(20,8)$, $(10,24),(21,9),(16,26),(27,14),(38,22),(35,2),(19,30),(3,31))$ $((29,7),(17,25),(22,15),(11,24),(6,19),(3,16),(12,21),(36,23),(27,33),(20,9),(28,13),(35,1)$, $(18,0),(26,34),(38,8),(10,31),(2,30),(5,39),(32,4),(14,37))$
$((39,11),(2,18),(7,28),(0,23),(25,10),(12,8),(33,5),(24,16),(27,36),(26,35),(15,9),(37,29)$, $(20,6),(4,34),(32,14),(17,21),(1,30),(31,13),(22,3),(38,19))$
$((9,27),(6,12),(25,8),(19,37),(34,21),(5,36),(3,39),(15,2),(35,18),(11,29),(30,26),(31,7)$, $(28,4),(20,10),(38,1),(14,22),(23,16),(33,13),(32,0),(17,24))$ $((34,0),(23,14),(21,6),(25,1),(32,10),(33,8),(27,13),(26,31),(28,12),(20,16),(17,35),(29,19)$, $(22,9),(11,36),(38,5),(4,24),(39,15),(2,37),(3,18),(30,7))$ $((26,13),(30,8),(15,0),(35,25),(38,2),(7,27),(33,4),(39,24),(9,14),(37,10),(11,20),(29,18)$, $(22,16),(17,3),(34,6),(31,19),(23,1),(12,32),(36,28),(21,5))$
$((36,10),(9,31),(28,14),(20,38),(25,6),(5,24),(17,33),(32,7),(8,23),(1,37),(21,4),(15,26)$, $(27,34),(18,39),(2,11),(30,16),(3,29),(12,22),(0,19),(35,13))$ $((3,35),(39,27),(16,6),(25,2),(23,15),(9,24),(29,38),(33,7),(36,14),(19,1),(0,26),(8,11)$, $(34,20),(30,17),(28,32),(18,4),(22,13),(37,12),(10,21),(31,5))$ $((10,34),(31,6),(3,37),(9,35),(27,18),(24,15),(2,29),(17,32),(39,16),(1,33),(5,22),(7,11)$, $(19,23),(4,12),(26,38),(36,0),(14,25),(30,20),(28,8),(21,13))$ $((0,25),(1,20),(2,13),(3,12),(4,29),(5,35),(6,26),(7,10),(8,32),(9,23),(11,37),(14,30)$, $(15,33),(16,36),(17,28),(18,21),(19,34),(22,39),(24,38),(27,31))$
$\alpha=8, \beta=14:$
$((14,2,31,24,9),(19,1,20,4,35),(17,37,27,8,39),(28,30,15,7,18),(22,10,33,6,16)$,
$(26,11,38,13,36),(23,32,0,21,3),(29,12,5,25,34))$
$((17,38,0,22,1),(4,30,10,20,14),(27,16,26,5,34),(6,24,19,32,25),(31,3,29,33,8)$,
$(28,2,37,13,9),(11,23,12,7,36),(15,21,39,18,35))$
$((15,1,29,37,24),(33,21,31,26,12),(23,38,22,13,7),(20,5,39,27,0),(30,2,10,28,11)$, $(36,14,34,4,17),(9,19,8,18,25),(3,16,32,6,35))$
$((12,37,11,1,24),(2,26,10,5,21),(16,7,22,17,8),(23,31,15,33,18),(14,3,34,19,27)$,
$(28,6,39,4,32),(0,36,29,35,25),(38,9,30,13,20))$
$((35,28,5,33,27),(36,6,12,38,8),(22,37,10,3,19),(34,18,20,15,9),(17,21,4,26,7)$,
$(32,2,16,39,13),(30,25,14,23,1),(24,11,31,29,0))$
$((24,39,29,2,33),(0,13,28,34,26),(5,11,3,22,36),(38,4,18,30,14),(15,23,6,17,32)$, $(25,19,21,9,10),(35,8,37,7,20),(1,12,27,31,16))$
$((32,10,6,14,7),(24,8,26,35,16),(19,36,9,33,4),(22,18,0,30,5),(28,31,13,23,37)$,
$(3,25,2,12,20),(17,34,21,11,27),(38,1,39,15,29))$
$((2,22,35,12,36),(5,15,34,0,23),(14,24,3,37,1),(32,18,6,21,8),(11,20,9,27,7)$,
$(33,25,39,26,13),(16,4,31,10,38),(30,29,17,28,19))$
$((6,19),(9,31),(30,8),(15,3),(26,37),(29,5),(21,16),(13,27),(22,11),(32,24),(35,10),(25,7)$, $(0,14),(23,39),(17,2),(4,12),(18,38),(1,34),(33,28),(36,20))$
$((26,18),(11,29),(13,3),(9,35),(28,38),(14,8),(34,2),(20,39),(19,23),(22,12),(10,24),(37,5)$, $(25,4),(27,1),(16,36),(6,15),(21,32),(30,7),(17,31),(33,0))$
$((27,2),(38,7),(4,36),(3,17),(28,15),(29,14),(19,26),(18,1),(11,25),(30,24),(13,8),(39,0)$, $(33,22),(23,35),(34,20),(37,6),(5,31),(16,9),(10,21),(32,12))$
$((21,18),(33,14),(16,28),(5,13),(39,9),(37,19),(17,26),(34,11),(4,23),(32,29),(6,22),(36,25)$, $(10,7),(15,0),(12,8),(31,1),(38,3),(20,2),(30,27),(24,35))$
$((34,10),(8,23),(13,1),(24,4),(11,35),(18,27),(28,39),(15,25),(6,31),(21,36),(20,37),(12,0)$, $(26,33),(2,38),(29,16),(22,14),(19,7),(32,5),(30,3),(9,17))$ $((12,25),(22,31),(10,23),(33,1),(19,39),(32,11),(35,0),(6,34),(30,21),(27,4),(15,8),(14,5)$, $(36,28),(17,20),(18,3),(13,2),(7,24),(38,26),(37,16),(29,9))$
$((7,35),(1,28),(11,4),(12,30),(2,24),(36,23),(38,5),(20,33),(17,25),(6,29),(22,39),(10,8)$, $(34,13),(14,21),(32,26),(18,9),(37,15),(31,19),(0,16),(27,3))$ $((6,11),(19,38),(24,17),(21,7),(14,28),(13,4),(18,36),(23,2),(39,10),(30,20),(31,0),(22,15)$, $(32,3),(25,37),(8,29),(35,1),(34,12),(26,9),(27,5),(16,33))$
$((27,36),(14,32),(8,11),(5,24),(33,7),(4,10),(21,37),(34,22),(30,16),(15,38),(23,9),(17,35)$, $(31,20),(0,19),(1,25),(39,2),(3,26),(18,29),(12,28),(6,13))$
$((14,35),(24,18),(22,8),(21,1),(20,19),(5,16),(6,38),(37,0),(32,27),(25,31),(28,4),(30,26)$, $(34,7),(15,2),(29,13),(3,33),(11,39),(17,23),(36,10),(9,12))$
$((12,3),(28,0),(26,15),(33,17),(20,16),(39,7),(30,22),(4,29),(21,38),(10,1),(18,31),(19,5)$, $(11,2),(32,9),(14,37),(25,8),(6,27),(23,34),(36,24),(35,13))$
$((0,17),(38,27),(24,34),(3,39),(32,20),(25,13),(22,9),(36,1),(15,4),(7,31),(26,14),(23,16)$, $(6,30),(29,10),(18,37),(33,11),(19,2),(12,21),(28,8),(35,5))$
$((11,0),(23,30),(39,14),(15,36),(37,9),(25,38),(8,20),(29,7),(12,31),(21,35),(17,5),(33,19)$, $(27,10),(3,28),(13,24),(34,16),(26,6),(4,22),(2,18),(1,32))$ $((0,10),(1,26),(2,35),(3,36),(4,37),(5,18),(6,20),(7,28),(8,34),(9,11),(12,39),(13,21)$, $(14,31),(15,27),(16,25),(17,30),(19,29),(22,32),(23,33),(24,38))$
$\alpha=9, \beta=12$ :
$((25,5,12,22,7),(6,35,21,11,32),(29,4,18,33,0),(24,3,39,28,8),(15,31,19,30,23)$,
$(16,1,27,14,34),(9,10,26,38,17),(2,36,13,20,37))$
$((23,38,0,26,18),(15,7,10,28,32),(29,13,22,9,34),(20,36,24,30,17),(31,1,35,12,8)$, $(3,21,5,39,11),(16,6,27,33,2),(19,4,14,25,37))$
$((8,14,23,2,18),(24,4,27,16,37),(36,11,20,38,19),(34,7,21,39,17),(15,22,30,6,29)$,
$(31,28,1,32,9),(0,13,26,33,12),(25,3,10,5,35))$
$((3,37,1,30,20),(12,6,39,2,32),(28,7,27,9,16),(17,25,13,4,33),(18,0,19,21,34)$,
$(5,38,8,26,14),(23,35,24,15,36),(31,29,10,22,11))$
$((21,8,22,31,0),(37,10,30,9,18),(16,35,19,28,5),(23,33,7,20,32),(13,38,27,2,24)$,
$(17,1,25,12,29),(4,36,14,39,15),(34,6,11,26,3))$
$((22,34,1,26,35),(15,30,3,23,5),(18,29,2,12,36),(28,14,21,33,6),(0,20,31,7,16)$,
$(10,25,9,24,38),(13,8,19,39,27),(17,37,11,4,32))$
$((35,28,9,29,14),(4,30,13,6,23),(25,11,38,15,8),(26,12,3,19,34),(39,20,33,24,1)$,
$(37,21,16,36,0),(32,7,18,27,5),(17,22,2,10,31))$
$((26,36,9,11,5),(33,1,18,31,13),(22,38,14,24,39),(27,0,25,6,15),(8,20,4,21,17)$,
$(16,29,37,12,30),(35,3,28,34,2),(10,23,7,19,32))$
$((13,34,27,35,7),(37,28,18,32,14),(4,17,26,39,12),(21,6,31,2,30),(11,23,8,36,29)$,
$(0,22,5,24,10),(19,1,15,20,9),(38,3,33,16,25))$
$((34,11),(30,18),(12,24),(4,10),(0,32),(16,22),(5,19),(36,17),(29,33),(25,15),(21,38),(8,35)$, $(2,28),(14,31),(9,26),(23,37),(27,3),(20,6),(7,39),(1,13))$
$((29,30),(12,7),(36,5),(4,16),(20,1),(11,35),(10,33),(3,18),(28,17),(38,2),(32,27),(21,13)$, $(8,39),(9,23),(0,14),(19,26),(34,24),(37,15),(6,22),(31,25))$
$((6,19),(38,4),(27,8),(25,2),(22,3),(0,15),(7,30),(18,5),(17,35),(29,39),(21,36),(26,37)$, $(9,14),(20,10),(31,24),(1,11),(33,28),(23,13),(34,12),(16,32))$
$((22,4),(33,11),(37,9),(18,35),(6,10),(14,2),(36,25),(27,30),(16,31),(15,26),(29,19),(20,5)$, $(3,17),(8,34),(38,12),(24,7),(32,21),(1,23),(28,13),(0,39))$
$((13,5),(29,32),(24,11),(22,1),(33,8),(30,28),(39,10),(14,3),(18,25),(31,12),(4,26),(20,16)$, $(15,35),(36,6),(34,0),(7,17),(21,2),(9,38),(19,23),(27,37))$ $((3,32),(31,23),(10,27),(11,7),(34,5),(38,29),(6,17),(39,16),(30,0),(22,36),(4,35),(15,28)$, $(37,8),(25,33),(21,12),(20,18),(2,26),(14,1),(9,13),(24,19))$ $((15,34),(7,14),(20,12),(32,25),(11,30),(31,3),(28,38),(9,33),(5,17),(16,24),(27,36),(26,6)$, $(13,37),(19,2),(1,21),(39,4),(10,35),(0,23),(29,8),(22,18))$ $((31,27),(15,33),(25,19),(17,0),(26,16),(18,38),(35,9),(24,6),(30,14),(8,11),(5,37),(4,28)$, $(12,1),(13,39),(21,10),(7,36),(2,20),(29,3),(34,23),(32,22))$ $((0,24),(22,14),(26,30),(15,9),(17,2),(23,12),(3,16),(25,4),(21,31),(36,1),(11,28),(32,13)$, $(39,18),(10,8),(34,20),(33,5),(29,35),(19,27),(7,37),(6,38))$ $((31,4),(5,29),(28,12),(19,33),(14,20),(30,8),(27,17),(35,13),(23,39),(36,3),(15,2),(22,37)$, $(6,18),(9,21),(26,7),(16,38),(25,34),(11,0),(32,24),(1,10))$
$((25,39),(9,12),(31,26),(18,21),(1,38),(33,22),(29,7),(27,11),(23,16),(5,30),(28,36),(37,4)$, $(0,35),(19,20),(15,3),(13,2),(24,17),(14,6),(34,10),(8,32))$
$((0,28),(1,29),(2,11),(3,13),(4,34),(5,31),(6,37),(7,38),(8,16),(9,39),(10,36),(12,27)$, $(14,33),(15,21),(17,23),(18,24),(19,22),(20,35),(25,30),(26,32))$
$\alpha=10, \beta=10$ :
$((37,2,23,7,12),(19,20,13,29,4),(25,11,1,16,5),(35,0,14,28,8),(15,21,30,6,32)$, $(18,38,10,24,34),(9,33,3,31,26),(27,36,17,22,39))$
$((0,15,22,30,25),(11,8,31,24,5),(7,32,3,19,33),(16,34,23,10,39),(38,6,37,18,20)$,
$(27,1,28,17,4),(29,2,14,26,36),(12,9,13,21,35))$
$((5,12,4,25,19),(10,30,14,38,28),(0,34,29,8,18),(6,20,15,39,21),(26,2,24,3,11)$,
$(1,36,9,16,32),(27,7,31,22,35),(37,17,23,33,13))$
$((17,20,12,36,21),(14,3,18,23,35),(31,29,5,32,27),(8,34,26,38,15),(6,11,24,1,22)$,
$(10,37,28,33,2),(0,16,30,4,39),(25,13,7,19,9))$
$((12,39,24,7,26),(37,16,4,20,14),(21,18,33,6,10),(11,31,19,28,30),(25,36,23,5,17)$,
$(13,8,27,2,22),(32,0,38,29,9),(3,15,35,1,34))$
$((20,34,22,16,3),(8,37,0,10,26),(29,7,14,24,32),(11,33,5,31,9),(28,35,18,39,13)$,
$(21,19,36,2,12),(27,17,30,1,38),(4,23,6,25,15))$
$((27,6,17,34,15),(21,16,25,18,7),(20,37,5,26,33),(8,22,4,14,39),(10,9,30,24,35)$, $(23,31,0,28,3),(1,29,19,38,12),(2,32,13,36,11))$
$((4,10,34,6,31),(14,23,11,28,32),(24,0,33,15,36),(5,38,16,8,20),(37,7,25,3,29)$,
$(1,18,26,35,13),(30,27,12,22,19),(9,17,39,2,21))$
$((10,3,12,34,27),(26,39,19,23,1),(35,11,0,17,7),(8,32,4,33,24),(28,16,6,29,15)$, $(14,22,9,37,21),(20,36,5,18,30),(31,13,38,2,25))$
$((36,16,24,18,22),(27,5,15,23,9),(7,20,11,29,39),(21,33,17,2,31),(13,30,3,35,4)$, $(14,8,38,25,34),(37,1,10,32,26),(6,28,12,0,19))$
$((11,38),(36,3),(22,5),(18,6),(8,19),(29,0),(28,4),(1,39),(23,37),(16,20),(15,24),(33,27)$, $(34,9),(2,30),(17,35),(13,26),(12,31),(25,14),(7,10),(21,32))$ $((10,5),(34,2),(21,0),(33,12),(30,8),(13,23),(32,22),(35,6),(15,37),(31,14),(27,3),(11,4)$, $(24,17),(19,26),(36,18),(9,28),(39,25),(1,20),(7,38),(16,29))$ $((35,16),(4,37),(21,1),(2,18),(17,31),(30,7),(25,32),(34,5),(36,8),(33,10),(29,14),(9,20)$, $(26,3),(19,27),(28,39),(0,13),(23,38),(24,12),(6,15),(22,11))$ $((36,10),(22,33),(25,1),(11,27),(32,17),(0,20),(26,30),(31,18),(4,38),(2,13),(16,23),(28,34)$, $(12,29),(5,35),(37,19),(15,7),(6,14),(24,9),(8,21),(39,3))$ $((27,13),(26,6),(8,10),(34,7),(12,25),(2,16),(30,5),(37,11),(4,18),(0,22),(36,28),(31,15)$, $(9,14),(3,21),(20,39),(33,1),(38,24),(23,32),(17,29),(19,35))$ $((3,37),(21,4),(11,7),(34,19),(32,12),(33,25),(0,26),(1,15),(8,17),(35,20),(27,16),(14,36)$, $(18,29),(2,28),(23,30),(5,13),(10,31),(9,39),(22,38),(6,24))$ $((9,18),(25,10),(23,39),(12,6),(32,20),(15,2),(37,27),(34,11),(24,19),(0,30),(13,3),(35,29)$, $(31,28),(16,26),(22,7),(14,1),(4,36),(38,17),(8,33),(21,5))$ $((5,39),(12,30),(25,37),(13,24),(9,38),(23,8),(26,15),(22,3),(29,10),(0,27),(2,35),(7,16)$, $(19,32),(31,20),(33,14),(17,1),(21,11),(6,36),(34,4),(18,28))$ $((15,9),(23,0),(35,25),(34,21),(36,7),(26,17),(38,3),(37,22),(5,28),(33,16),(20,10),(18,32)$, $(13,6),(14,27),(29,30),(19,2),(4,24),(11,39),(8,12),(1,31))$ $((35,9),(18,27),(19,1),(28,7),(13,34),(17,3),(25,8),(24,37),(10,22),(2,20),(0,36),(5,14)$, $(4,26),(6,39),(23,12),(38,21),(11,32),(15,30),(33,29),(31,16))$
$\alpha=11, \beta=8:$
$((21,18,24,5,12),(39,3,11,28,0),(36,19,4,27,15),(17,33,7,32,8),(37,14,23,38,22)$,
$(13,26,10,9,31),(1,25,34,2,35),(20,30,16,29,6))$ $((35,8,36,22,12),(0,11,2,26,31),(14,38,6,33,5),(18,29,34,1,20),(28,16,7,27,37)$, $(39,17,32,13,21),(30,4,24,15,3),(9,25,10,23,19))$
$((0,24,32,11,38),(17,5,28,33,21),(37,1,14,31,16),(8,20,19,35,27),(15,26,12,2,22)$,
$(29,10,6,23,30),(4,18,36,3,13),(25,7,34,9,39))$
$((33,19,30,10,8),(32,25,36,9,28),(21,37,17,2,14),(24,34,20,4,16),(18,35,7,11,27)$,
$(29,12,3,38,1),(26,0,15,23,39),(13,5,22,31,6))$
$((39,14,6,37,2),(1,21,10,27,36),(15,35,16,3,28),(24,13,9,22,17),(30,26,19,29,11)$,
$(4,38,7,31,23),(34,0,33,18,8),(5,20,32,12,25))$
$((4,36,24,19,34),(23,37,8,12,7),(16,27,1,13,20),(18,3,29,31,28),(6,26,9,14,32)$,
$(11,21,35,0,22),(33,2,30,17,25),(38,10,39,15,5))$
$((12,27,19,25,4),(2,36,23,33,29),(21,0,16,1,32),(18,7,39,22,6),(37,9,15,31,20)$,
$(35,24,10,5,11),(30,8,14,28,13),(26,34,3,17,38))$
$((27,9,12,20,0),(30,22,16,5,18),(28,17,1,39,6),(33,14,26,37,15),(8,13,38,25,31)$,
$(19,3,23,34,21),(24,7,29,36,11),(2,32,4,35,10))$
$((31,18,2,28,19),(8,23,13,27,39),(20,7,22,4,17),(25,30,5,35,6),(9,16,21,15,38)$,
$(10,36,26,32,0),(3,24,12,1,33),(14,34,11,37,29))$
$((29,0,23,18,32),(35,13,25,11,9),(37,7,10,33,4),(1,22,34,12,28),(21,6,15,20,3)$,
$(26,8,16,36,17),(31,2,19,39,5),(14,30,27,38,24))$
$((16,32,27,14,25),(37,10,1,23,5),(7,30,21,4,26),(31,3,22,33,24),(34,13,29,35,17)$,
$(39,11,20,9,18),(38,28,8,15,2),(19,0,12,36,6))$
$((12,23),(35,25),(38,20),(27,31),(17,0),(3,14),(1,26),(39,13),(18,34),(21,5),(37,19),(33,9)$, $(29,15),(10,32),(6,11),(16,2),(28,4),(8,22),(24,30),(7,36))$
$((33,11),(26,16),(4,31),(9,30),(21,36),(32,5),(35,14),(37,0),(18,25),(20,39),(12,38),(7,15)$, $(22,13),(2,23),(24,1),(34,6),(10,28),(17,29),(3,27),(19,8))$
$((22,14),(18,0),(25,37),(13,7),(10,31),(9,21),(30,1),(26,11),(19,38),(2,27),(33,20),(35,28)$, $(24,8),(23,17),(32,3),(39,16),(36,5),(12,6),(29,4),(15,34))$
$((34,27),(2,20),(8,25),(12,37),(18,1),(21,7),(11,31),(3,26),(6,17),(0,14),(4,39),(36,13)$, $(32,15),(30,28),(5,19),(24,9),(10,22),(33,16),(35,23),(29,38))$ $((39,29),(31,17),(16,6),(20,10),(2,24),(25,3),(1,11),(5,27),(32,19),(30,12),(15,4),(21,8)$, $(22,35),(7,14),(36,0),(38,18),(13,37),(34,28),(9,23),(33,26))$ $((27,33),(39,28),(32,23),(29,9),(7,17),(3,37),(4,10),(0,25),(5,26),(12,31),(14,36),(11,8)$, $(15,30),(6,24),(34,16),(22,18),(13,2),(20,35),(19,1),(21,38))$ $((10,3),(7,28),(9,32),(38,16),(18,37),(21,31),(20,36),(6,30),(0,13),(14,4),(1,15),(11,23)$, $(2,25),(19,22),(33,12),(17,27),(26,35),(29,8),(24,39),(34,5))$ $((34,10),(38,8),(25,15),(0,30),(3,35),(2,21),(5,29),(20,14),(32,22),(24,37),(16,23),(31,1)$, $(39,12),(33,13),(4,11),(26,18),(27,6),(17,9),(28,36),(7,19))$
$\alpha=12, \beta=6$ :
$((32,24,36,8,28),(37,20,1,30,17),(39,13,33,11,3),(34,22,5,31,0),(35,25,19,27,12)$, $(10,6,38,15,29),(9,14,21,2,16),(18,4,26,7,23))$
$((29,16,31,24,12),(18,25,39,14,32),(1,38,17,5,27),(8,34,26,9,19),(20,10,35,22,36)$,
$(28,3,30,15,4),(7,37,11,0,33),(2,23,6,21,13))$
$((22,19,28,15,1),(30,25,37,24,6),(12,7,39,26,5),(10,3,34,14,2),(32,21,36,29,13)$,
$(33,20,35,27,4),(8,38,16,23,11),(18,0,17,9,31))$ $((13,35,1,33,9),(5,14,30,21,16),(2,25,8,10,39),(26,12,31,22,18),(17,6,15,34,28)$,
$(36,4,37,29,0),(38,23,32,19,20),(11,7,27,3,24))$
$((35,16,25,12,9),(15,2,36,17,26),(14,20,4,13,24),(30,22,7,31,29),(0,21,19,38,27)$,
$(28,6,37,1,11),(33,18,5,34,10),(39,8,32,3,23))$ $((3,22,12,4,35),(13,36,25,1,31),(26,8,20,18,38),(28,0,19,7,30),(2,17,24,16,33)$,
$(32,6,27,34,11),(21,37,14,29,39),(9,15,5,10,23))$
$((13,23,34,7,25),(1,29,33,19,36),(38,9,22,8,12),(10,21,18,2,26),(37,28,39,17,3)$,
$(35,6,16,32,0),(15,31,14,4,24),(11,27,30,5,20))$
$((38,24,5,37,10),(14,3,16,26,1),(32,27,13,6,12),(36,18,7,35,28),(39,20,9,29,11)$,
$(19,31,17,23,4),(22,33,25,15,0),(21,8,30,2,34))$ $((5,33,14,0,38),(9,37,18,35,11),(22,4,30,12,2),(1,28,31,21,17),(39,15,7,16,27)$,
$(24,10,32,25,34),(8,13,26,36,23),(19,6,20,3,29))$
$((14,25,0,10,28),(32,26,31,27,9),(20,13,3,36,7),(22,6,18,1,16),(5,39,12,21,11)$,
$(34,29,2,38,4),(37,15,33,17,8),(24,19,35,23,30))$
$((37,0,12,1,23),(9,34,18,24,39),(30,10,4,31,20),(36,14,38,11,6),(22,13,7,32,15)$, $(5,35,29,17,25),(26,19,3,21,33),(2,27,8,16,28))$ $((10,22,14,27,36),(16,30,0,39,4),(20,32,29,8,15),(31,6,26,11,2),(19,23,5,13,37)$, $(7,21,9,25,38),(33,28,18,3,12),(35,24,1,34,17))$
$((38,13),(2,19),(8,24),(1,21),(22,37),(25,31),(7,29),(35,26),(12,28),(9,10),(5,32),(17,27)$,
$(11,4),(6,39),(18,30),(36,15),(16,34),(33,3),(23,14),(20,0))$
$((9,18),(5,29),(24,2),(35,14),(8,33),(3,15),(26,0),(30,11),(7,10),(25,4),(1,32),(38,21)$, $(16,36),(13,28),(34,6),(20,12),(23,31),(37,27),(22,17),(19,39))$
$((25,11),(36,12),(17,4),(20,16),(13,34),(35,2),(9,28),(23,15),(3,38),(32,22),(19,30),(6,33)$, $(0,24),(26,37),(21,5),(18,29),(1,39),(8,31),(14,7),(10,27))$
$((39,18),(12,34),(26,3),(5,28),(31,11),(37,16),(13,30),(22,38),(36,9),(24,7),(10,25),(35,8)$, $(17,32),(21,15),(2,20),(23,0),(19,1),(6,14),(33,27),(4,29))$ $((6,25),(8,14),(18,27),(21,4),(16,39),(9,24),(13,0),(2,32),(23,33),(30,26),(36,5),(35,15)$, $(3,31),(38,29),(12,37),(17,20),(22,11),(7,28),(34,19),(10,1))$ $((34,20),(10,31),(38,28),(37,2),(1,13),(35,21),(18,8),(19,5),(15,27),(11,36),(23,12),(24,33)$, $(3,25),(39,22),(17,7),(4,32),(14,26),(6,29),(0,16),(30,9))$
$\alpha=13, \beta=4$ :
$((2,39,4,13,29),(30,23,11,38,8),(34,15,35,3,28),(33,5,16,1,24),(27,32,9,25,7)$,
$(17,22,31,19,21),(14,36,20,18,26),(6,10,0,37,12))$
$((1,38,29,30,26),(20,35,8,13,33),(32,21,7,24,10),(3,16,31,23,37),(27,34,14,39,6)$,
$(5,22,12,0,15),(2,19,36,25,18),(9,17,28,4,11))$
$((27,16,21,5,18),(38,24,6,20,15),(36,0,25,1,22),(13,31,3,23,39),(7,26,11,32,28)$,
$(29,33,12,35,19),(37,10,8,14,2),(34,4,17,30,9))$
$((3,32,5,14,21),(39,25,19,38,26),(24,0,34,1,17),(6,37,16,20,13),(23,10,27,12,8)$,
$(4,30,28,18,33),(2,35,29,31,11),(36,7,22,15,9))$
$((1,15,37,21,18),(35,7,32,23,6),(0,22,39,10,31),(2,13,34,20,38),(19,26,12,9,28)$,
$(24,5,27,11,30),(16,36,29,17,8),(14,3,33,25,4))$
$((22,19,33,17,2),(23,36,21,13,5),(12,20,8,32,1),(10,29,34,24,9),(35,25,30,15,28)$,
$(6,38,14,31,26),(27,0,11,3,39),(4,37,18,7,16))$
$((16,35,5,34,22),(17,36,10,38,25),(39,1,19,30,0),(11,33,23,12,28),(4,26,8,27,31)$,
$(37,24,2,32,13),(14,6,21,15,7),(3,20,9,29,18))$
$((0,21,12,34,23),(25,5,37,28,10),(27,33,15,3,19),(8,18,31,20,39),(22,14,29,4,32)$,
$(38,9,35,17,7),(2,26,13,1,30),(16,6,36,11,24))$
$((35,4,24,14,1),(37,7,13,0,19),(39,16,32,26,9),(27,36,8,22,38),(29,5,28,2,12)$,
$(30,20,17,23,18),(21,34,3,10,33),(15,6,11,25,31))$
$((29,15,23,19,8),(2,16,34,17,27),(6,30,21,9,33),(7,31,5,12,39),(26,10,4,22,37)$,
$(11,1,20,14,35),(18,38,3,36,24),(0,32,25,13,28))$
$((38,21,10,7,23),(28,16,26,33,1),(9,22,30,27,14),(24,12,3,13,35),(34,25,37,11,8)$,
$(36,2,15,39,5),(4,18,32,20,19),(31,17,0,29,6))$
$((0,14,23,2,20),(1,10,5,11,21),(3,17,6,32,24),(4,12,30,13,38),(7,29,39,19,34)$,
$(8,25,15,27,37),(9,16,33,28,31),(18,22,35,26,36))$
$((0,16,23,9,18),(1,27,4,15,36),(2,10,35,21,31),(3,25,12,32,29),(5,19,6,28,38)$,
$(7,20,37,14,30),(8,24,13,22,33),(11,34,26,17,39))$
$((0,26),(1,23),(2,21),(3,22),(4,20),(5,17),(6,18),(7,33),(8,15),(9,37),(10,34),(11,29)$,
$(12,38),(13,36),(14,25),(16,30),(19,32),(24,31),(27,35),(28,39))$
$((0,33),(1,29),(2,25),(3,26),(4,21),(5,20),(6,22),(7,11),(8,31),(9,19),(10,30),(12,36)$,
$(13,27),(14,28),(15,32),(16,38),(17,37),(18,34),(23,35),(24,39))$
$((0,35),(1,31),(2,33),(3,27),(4,23),(5,30),(6,34),(7,12),(8,21),(9,13),(10,20),(11,22)$,
$(14,32),(15,26),(16,25),(17,38),(18,39),(19,24),(28,36),(29,37))$
$((0,38),(1,37),(2,34),(3,30),(4,36),(5,26),(6,25),(7,19),(8,28),(9,27),(10,22),(11,20)$,
$(12,31),(13,23),(14,33),(15,24),(16,29),(17,32),(18,35),(21,39))$
$\alpha=14, \beta=2$ :
$((34,1,37,14,29),(19,5,31,3,23),(0,11,6,21,35),(27,15,38,17,2),(4,26,30,22,33)$,
$(9,13,39,25,36),(32,10,20,18,24),(16,28,7,12,8))$
$((24,39,22,17,33),(34,8,35,19,2),(4,13,6,28,30),(9,38,3,32,27),(16,5,12,1,21)$,
$(7,20,31,0,10),(37,23,18,26,15),(11,25,14,36,29))$
$((38,19,34,27,14),(12,31,24,15,23),(8,28,0,30,21),(35,2,26,1,29),(5,11,33,3,22)$,
$(37,13,20,36,10),(6,25,16,9,17),(7,32,4,39,18))$
$((8,24,4,14,33),(30,12,22,37,6),(36,15,39,0,26),(2,32,29,16,23),(19,7,31,28,9)$,
$(35,25,10,1,13),(11,21,18,38,27),(34,3,17,5,20))$
$((11,20,32,18,3),(37,12,4,36,17),(7,38,13,29,39),(14,5,25,33,1),(8,30,15,28,19)$,
$(9,24,6,23,35),(26,31,16,27,10),(21,34,0,22,2))$
$((20,37,2,25,17),(9,26,6,29,18),(5,23,11,4,28),(7,16,39,12,35),(13,21,15,1,30)$,
$(8,10,34,24,38),(0,32,22,19,33),(3,36,27,31,14))$
$((31,9,14,6,15),(37,28,38,1,24),(2,29,7,26,39),(5,34,12,20,33),(8,32,21,36,13)$,
$(4,18,22,10,23),(25,19,27,17,0),(3,16,30,11,35))$
$((11,8,23,14,39),(30,5,26,17,7),(4,31,21,9,15),(36,0,18,34,22),(28,32,12,24,2)$,
$(20,19,3,37,16),(38,29,10,35,6),(25,1,27,33,13))$
$((29,31,13,3,15),(5,37,18,1,32),(36,6,10,24,11),(38,23,30,19,26),(4,27,8,39,20)$,
$(16,35,17,21,0),(28,33,12,2,14),(7,22,9,25,34))$
$((21,38,4,34,14),(18,27,7,23,36),(26,12,29,0,37),(6,22,1,19,31),(3,10,2,13,28)$,
$(30,20,8,15,25),(9,33,16,32,11),(35,24,17,39,5))$
$((14,24,13,26,32),(29,5,36,8,17),(16,2,33,23,1),(30,9,20,35,27),(28,12,25,3,39)$,
$(19,37,7,15,0),(34,6,18,31,11),(4,22,38,10,21))$
$((0,12,3,20,14),(1,11,2,15,35),(4,10,5,24,19),(6,27,37,21,39),(7,25,38,16,36)$,
$(8,22,13,23,31),(9,29,30,17,32),(18,28,34,26,33))$
$((0,13,27,39,23),(1,20,6,12,36),(2,30,18,5,38),(3,21,33,7,24),(4,16,26,14,35)$,
$(8,25,32,19,29),(9,10,28,11,37),(15,22,31,17,34))$
$((0,20,15,5,27),(1,17,4,25,31),(2,18,35,28,36),(3,26,8,37,29),(6,19,39,10,33)$,
$(7,11,38,12,21),(9,23,32,13,34),(14,22,16,24,30))$ $((0,24),(1,39),(2,31),(3,27),(4,37),(5,21),(6,16),(7,13),(8,14),(9,12),(10,30),(11,22)$, $(15,32),(17,28),(18,25),(19,36),(20,38),(23,34),(26,35),(29,33))$
$((0,38),(1,28),(2,20),(3,30),(4,29),(5,13),(6,32),(7,14),(8,18),(9,39),(10,31),(11,26)$,
$(12,27),(15,33),(16,34),(17,23),(19,21),(22,35),(24,36),(25,37))$
$K_{3(4)}$ Let the vertex set be $\mathbb{Z}_{12}$ with vertex partition $\{0,1,2,3\},\{4,5,6,7\}$, and $\{8,9,10,11\}$.
$C_{3}$-factors: $\quad((0,6,11),(1,7,8),(2,5,10),(3,4,9))$
$((0,7,9),(1,6,10),(2,4,11),(3,5,8))$
$C_{4}$-factors: $\quad((0,4,1,5),(2,8,6,9),(3,10,7,11))$
$((0,8,4,10),(1,9,5,11),(2,6,3,7))$


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